FM分光とPDH法

（裏のテーマ）原子気体の偏光分光信号でレーザー線幅狭窄化は可能か？

2011.11.30　ランチミーティング

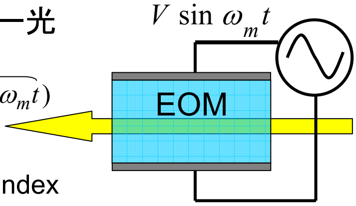
担当：鳥井

位相変調（PM）=周波数変調（FM）

位相変調されたレーザー光

$$E(t)=E_0e^{i\overbrace{(\omega_c t+M\sin\omega_m t)}^{\theta(t)}}$$

Modulation index
（変調指数）



$$E(t)=E_0e^{i\omega_c t}$$

「瞬間的（instantaneous）周波数」という概念を導入する

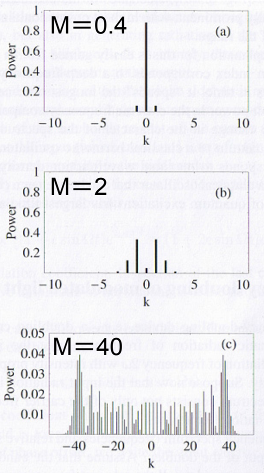
$$\omega(t)\equiv\frac{d\theta(t)}{dt}=\omega_C+\boxed{M\omega_m}\cos\omega_m t$$

modulation depth（周波数変調幅）

位相をsinで変調することは周波数をcosで変調することに相当

位相変調された光のスペクトル

サイドバンドの発生



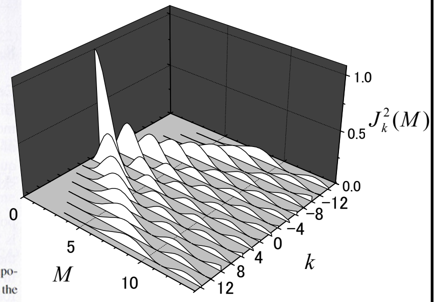
素朴な周波数変調に近づく

FIG. 8.3 Power spectrum of frequency-modulated field. The height of the k-th frequency component is given by $J_k^2(m)$. (a) $m = 0.4$, (b) $m = 2$, (c) $m = 40$ (note the difference in both the vertical and horizontal scales).

Budker, Kimball, DeMille, *Atomic Physics* (2004)

$$E_0 e^{i(\omega_c t + M\sin\omega_m t)}$$

$$= E_0 e^{i\omega_c t}\sum_{k=-\infty}^{\infty} J_k(M) e^{ik\omega_m t}$$



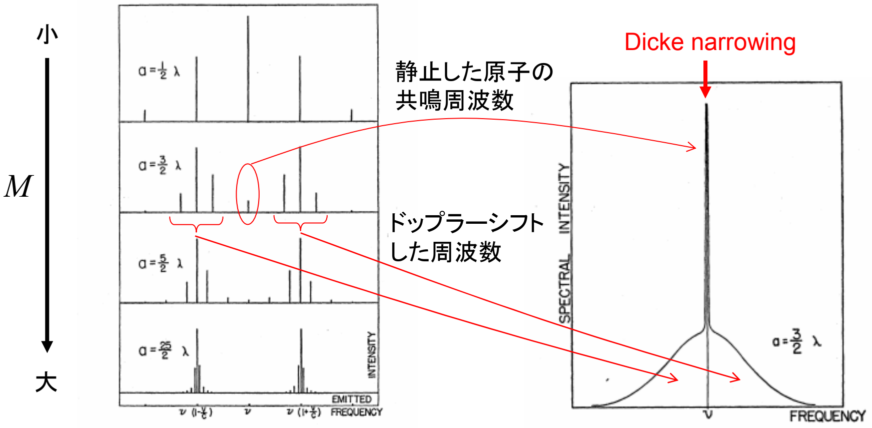
PHYSICAL REVIEW VOLUME 89, NUMBER 2 JANUARY 15, 1953

The Effect of Collisions upon the Doppler Width of Spectral Lines

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received September 17, 1952)

Quantum mechanically the Doppler effect results from the recoil momentum changing the translational energy of the radiating atom. The assumption that the recoil momentum is given to the radiating atom is shown to be incorrect if collisions are taking place. If the collisions do not cause broadening by affecting the internal state of the radiator, they result in a substantial narrowing of the Doppler broadened line.



小

M

大

Dicke narrowing

静止した原子の共鳴周波数

ドップラーシフトした周波数

FIG. 1. Spectral distribution of radiation emitted by an atom confined to a one-dimensional box of width a.

FIG. 2. Doppler broadened line of a gas in a one-dimensional box.

ベッセル関数など知らなくてよい

以降の議論では、常に$M<<1$を仮定する

$$E_0 e^{i(\omega_c t + M\sin\omega_m t)} = E_0 e^{i\omega_c t} e^{iM\sin\omega_m t}$$

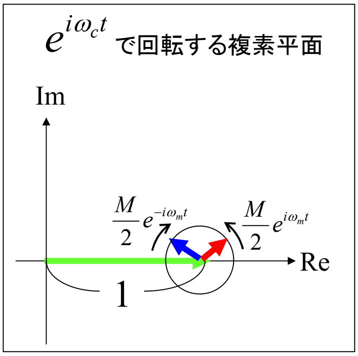
$$\cong E_0 e^{i\omega_c t}\left(1 + iM\sin\omega_m t\right)$$

$$= E_0 e^{i\omega_c t}\left(1 + iM\frac{e^{i\omega_m t} - e^{-i\omega_m t}}{2i}\right)$$

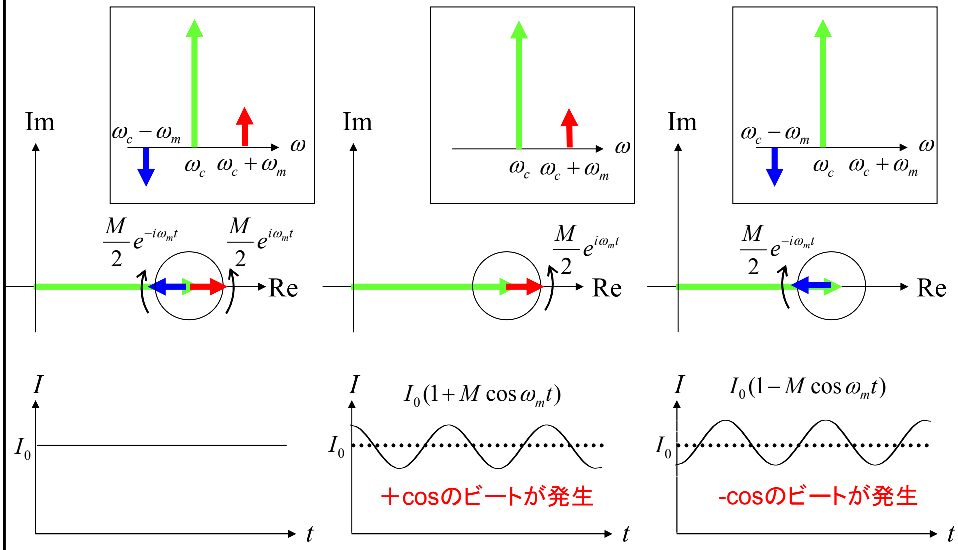
$$= E_0\left(e^{i\omega_c t} + \frac{M}{2}e^{i(\omega_c+\omega_m)t} - \frac{M}{2}e^{i(\omega_c-\omega_m)t}\right)$$

$$= E_0 e^{i\omega_c t}\left(\boxed{1} + \boxed{\frac{M}{2}e^{i\omega_m t}} - \boxed{\frac{M}{2}e^{-i\omega_m t}}\right)$$

反時計回り　時計回り



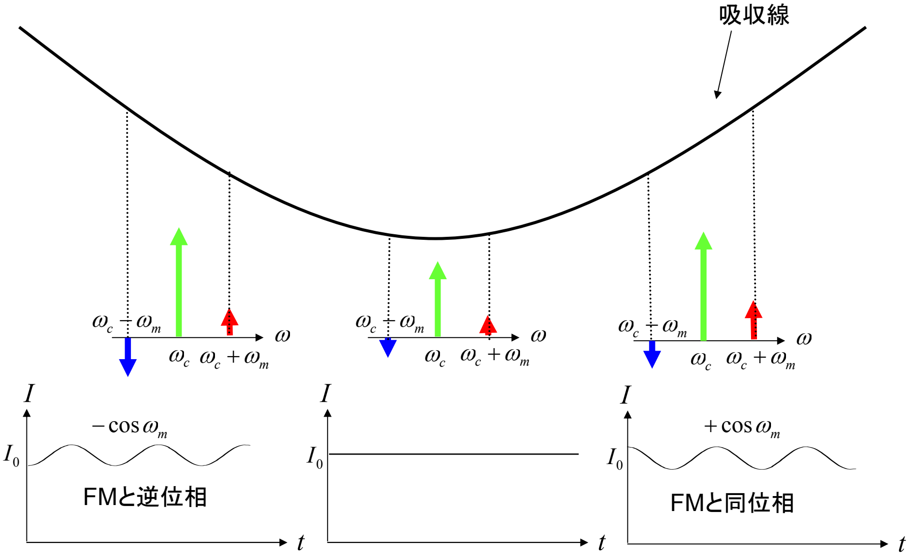
サイドバンドのバランスが崩れると



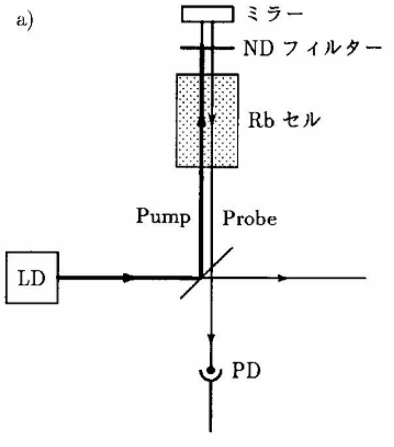
＋cosのビートが発生

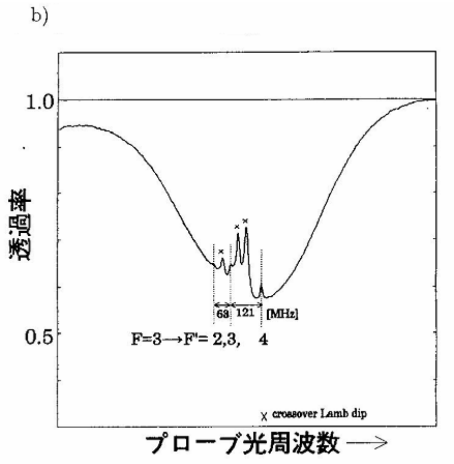
-cosのビートが発生

FM分光による吸収線の微分



Rb原子気体の飽和分光（自然幅Γ = 6MHz）





鳥井寿夫　修士論文（1994）

ナイーブなFM分光(吸収線の微分を見る）
M～100（$M\omega_m$=1MHz >>ω_m～10 kHz）

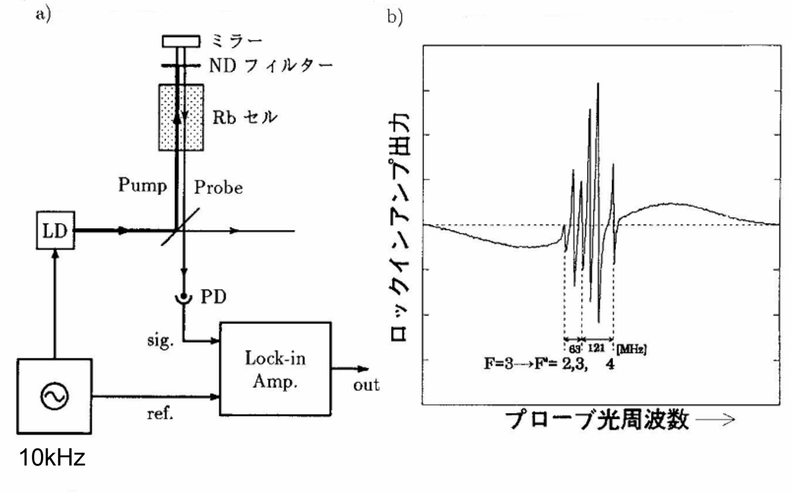
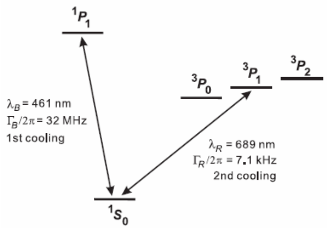
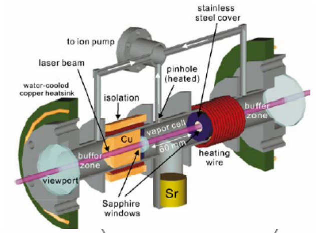


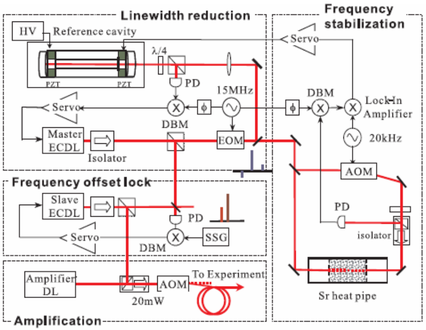
図 3.6: FM 変調法による^{85}Rb 原子 $F = 3 \rightarrow F'$遷移の飽和吸収スペクトル

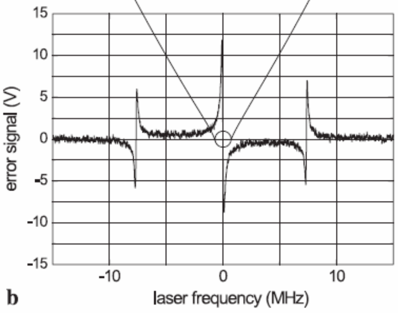
鳥井寿夫 修士論文（1994）

自然幅より大きい周波数変調
M～1（ω_m=15MHz >>Γ=7.1 kHz ）



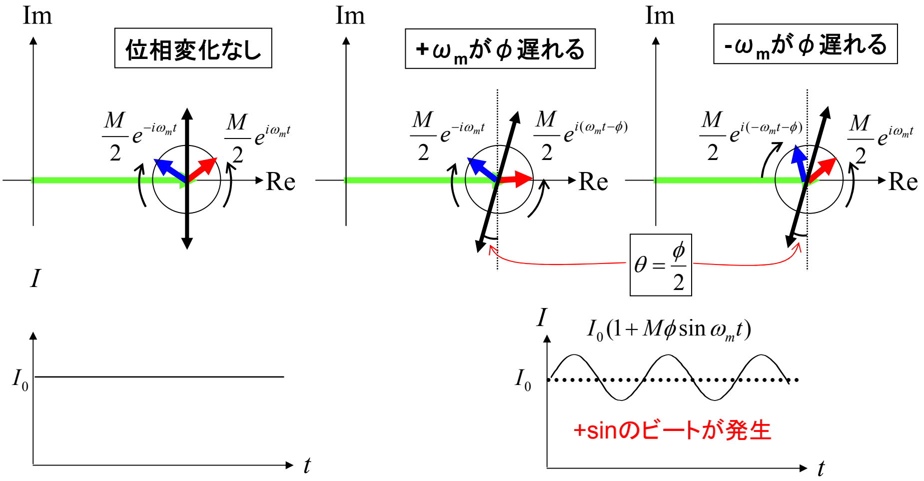






Li, et. al., Appl. Phys. B 78, 315 (2004)

サイドバンドの位相が変化すると



位相変化なし

+ω$_m$がφ遅れる

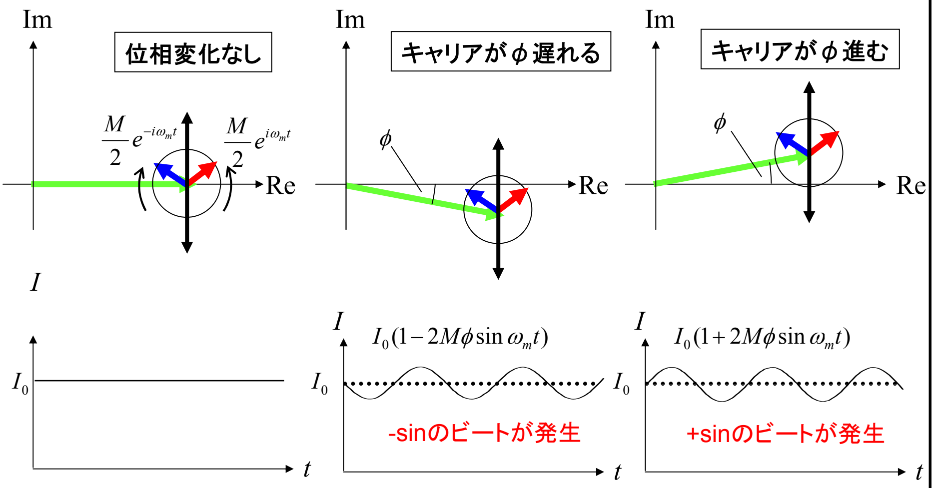
-ω$_m$がφ遅れる

$\theta=\frac{\phi}{2}$

$I_0(1+M\phi\sin\omega_m t)$

+sinのビートが発生

キャリアの位相がシフトすると



位相変化なし

キャリアがφ遅れる

キャリアがφ進む

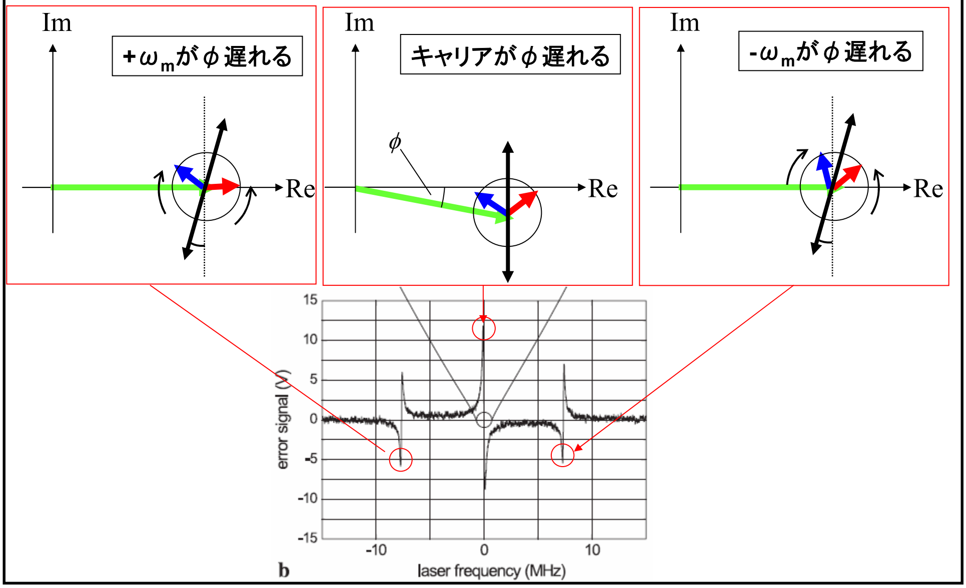
$I_0(1-2M\phi\sin\omega_m t)$

-sinのビートが発生

$I_0(1+2M\phi\sin\omega_m t)$

+sinのビートが発生

FM分光の分散モード



PDH法とFM分光の歴史

1947	マイクロ波のPound Stabilizer Rev. Sci. Instrum. **17**, 490 (1946)
1979	Bjorklund IBMでFM分光を発明？ Drever, Hall 色素レーザー線幅<100Hz
1980	Bjorklund FM分光論文 Opt. Lett. **5**, 15 (1980) Hansch-Couillaud法 Opt. Comm. **35**, 441 (1980)
1981	Hall FM飽和分光 Appl. Phys. Lett. **39**, 680 (1981)
1983	Drever,Hall PDH法に関する最初の論文 Appl. Phys. B. **31**, 97 (1983) Bjorklund FM分光の解説論文 Appl. Phys. B. **32**, 145 (1983) 本日配布した論文

January 1980 / Vol. 5, No. 1 / OPTICS LETTERS 15

Frequency-modulation spectroscopy: a new method for measuring weak absorptions and dispersions

Gary C. Bjorklund

IBM Research Laboratory, San Jose, California 95193

Received September 17, 1979

A new type of wavelength-modulation laser spectroscopy is accomplished by utilizing an external phase modulator driven at radio frequencies large compared to the width of the spectral feature of interest. The spectral feature is probed by a single frequency-modulated (FM) sideband, and the associated absorption and dispersion are measured by monitoring the resulting radio-frequency beat signal. Experimental results are presented for the measurement of Fabry–Perot resonances, I_2 vapor absorption lines, and saturation holes in Na vapor.

$$\tilde{E}_1(t) = E_0 \exp(i\omega_c t)$$

$$E_2(t) = \frac{E_0}{2} \sum_{n=-\infty}^{\infty} J_n(M) \exp[i(\omega_c + n\omega_m)t] + \text{c.c.}$$

吸収の差 位相の差

$$I_3(t) = \frac{cE_0{}^2}{8\pi} e^{-2\bar{\delta}}[1 - \Delta\delta M \cos\omega_m t + \Delta\phi M \sin\omega_m t]$$

最初のPDH信号

A Fabry–Perot resonator used in the reflection mode was employed to test the line-shape theory in the FM spectroscopy limit.

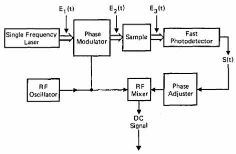


Fig. 1. Typical experimental arrangement for FM spectroscopy.

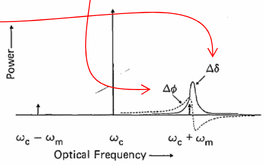


Fig. 2. Frequency-domain illustration of FM spectroscopy.

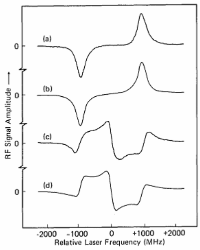


Fig. 3. Experimental and theoretical line shapes for FM spectroscopy of a Fabry–Perot resonance: (a) experimental in-phase signal, (b) theoretical in-phase signal, (c) experimental quadrature signal, (d) theoretical quadrature signal.

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680 Appl. Phys. Lett. **39**(9), 1 November 1981 0003-6951/81/210680–03$00.50

Optical heterodyne saturation spectroscopy

J. L. Hall,[a)] L. Hollberg, T. Baer, and H. G. Robinson

Joint Institute for Laboratory Astrophysics National Bureau of Standards and University of Colorado Boulder, Colorado 80309

(Received 26 May 1981; accepted for publication 18 August 1981)

We discuss a refined, hybrid rf/optical technique for studying sub-Doppler saturated absorption/dispersion resonances with excellent precision and symmetry. Sensitivity is limited mainly by fundamental noise in the signal. Resonance profiles obtained in I_2 are in remarkable agreement with theory. The method promises a new level of accuracy for laser locking to an optical resonance.

A very powerful technique developed for NMR is FM sideband spectroscopy,[5] which Bjorklund[6] has recently used in the optical domain via rf electro-optic phase modulation. Such techniques were also developed independently by us to servo-lock a tunable laser to a high finesse optical cavity,[7] following a suggestion by Drever.[8] Here we

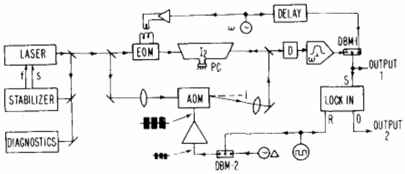


FIG. 1. Optical Heterodyne Saturation Spectrometer.

The signal-bearing probe beam is detected by a fast photodiode (D) whose output is filtered for the rf component at frequency ω and applied to the signal port of an rf doubly-balanced mixer. The rf reference signal is phase shifted by an adjustable delay line. The dc output of DBM-1 may be further processed by a lock-in amplifier to recover the signal (output 2) synchronous with the saturation chopping.

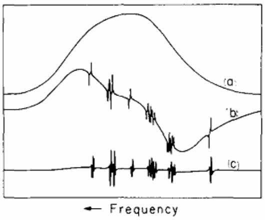


FIG. 2. Broad-scan Spectral Profiles. (a) Fluorescence from exciting the I_2 589.214-nm line by the probe beam alone. Overall sweep width about 2 GHz. (b) Output of rf doubly-balanced mixer with reference phase set to recover saturated absorption signals. (c) Same as (b) except rf phase set for dispersion. Note absence of Doppler background even without saturation chopping.

reference

[6] G. C. Bjorklund, Opt. Lett. **5**, 15 (1980).
[7] R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, and A. J. Munley, September 1979. A < 100-Hz dye laser linewidth was obtained!
[8] R. W. P. Drever (private communication).
[9] R. V. Pound, Rev. Sci. Instrum **17**, 490 (1946).

Appl. Phys. B 31, 97–105 (1983)

Applied Physics B Photophysics and Laser Chemistry

Laser Phase and Frequency Stabilization Using an Optical Resonator

R. W. P. Drever

University of Glasgow and California Institute of Technology, Pasadena, CA 91125, USA

J. L. Hall* and F. V. Kowalski**

Joint Institute for Laboratory Astrophysics, National Bureau of Standards and University of Colorado, Boulder, CO 80309, USA

J. Hough, G. M. Ford, A. J. Munley, and H. Ward

Department of Natural Philosophy, University of Glasgow, Glasgow, Scotland G12800

reference

4. Yu V. Troitskii: Sov. J. Quant. Electron. **8**, 628–631 (1978). This is apparently the first publication analyzing the advantages of reflection-mode operation of a cavity-type laser stabilizer

12. FM sideband methods in the optical domain were developed independently by G. C. Bjorklund: Opt Lett. **5**, 15–17 (1980) and used for spectroscopy by G. C. Bjorklund, M. D. Levenson: Phys. Rev. A **24**, 166–169 (1981); and by J. L. Hall, L. Hollberg, T. Baer, H. G. Robinson: Appl. Phys. Lett. **39**, 680–682 (1981) and in *Laser Spectroscopy V*, ed. by A. R. W. McKellar, T. Oka, B. P. Stoicheff (Springer, Berlin, Heidelberg, New York 1981) pp. 15–24

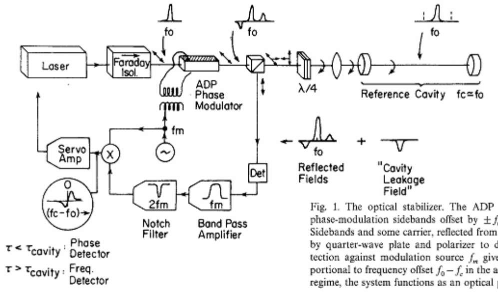


Fig. 1. The optical stabilizer. The ADP phase modulator produces phase-modulation sidebands offset by $\pm f_m$ from carrier frequency f_c. Sidebands and some carrier, reflected from reference cavity, are steered by quarter-wave plate and polarizer to detector. Phase-sensitive detection against modulation source f_m gives bipolar error signal proportional to frequency offset $f_0 - f_c$ in the adiabatic regime. In transient regime, the system functions as an optical phase detector (see text)

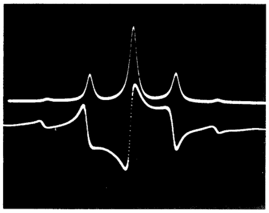
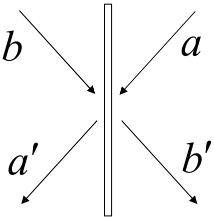


Fig. 2. Signals in the optical stabilizer. Upper curve, transmitted laser intensity as cavity is tuned over sideband structure of phase-modulated laser beam. First and second sidebands visible symmetrically on both sides of the carrier (prominent central line). Lower curve, output of the phase detector with phase reference adjusted for dispersion. Lock point is the zero in the central high-slope region. Note that the error signal sign is correct for servo purposes over the full spectral interval $2f_m$

ビームスプリッターはユニタリー変換



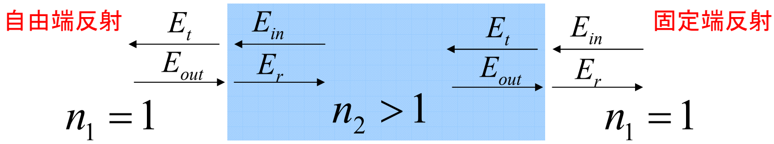
$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} t & re^{i\delta} \\ -re^{-i\delta} & t \end{pmatrix}}_{U} \begin{pmatrix} a \\ b \end{pmatrix}$$

$t^2 + r^2 = 1$ ならば $UU^+ = I$

$$\begin{pmatrix} E_t \\ E_r \end{pmatrix} = \begin{pmatrix} t & \ominus r \\ r & t \end{pmatrix} \begin{pmatrix} E_{in} \\ E_{out} \end{pmatrix}$$

$$r = \frac{n_2 - n_1}{n_1 + n_2}, \; t = \frac{2n_1 n_2}{n_1 + n_2}$$

$$\begin{pmatrix} E_t \\ E_r \end{pmatrix} = \begin{pmatrix} t & r \\ \ominus r & t \end{pmatrix} \begin{pmatrix} E_{in} \\ E_{out} \end{pmatrix}$$



Fabry-Perot共振器の複素反射率

共振器を一周してきた光の位相

$$\delta = -k \cdot 2L = -\frac{\omega}{\nu_{FSR}}$$

$$\nu_{FSR} \equiv \frac{c}{2L}$$

共振器内電場（右向きの進行波）

$$E_{cav} = \sum_{n=1}^{\infty} E_n = \frac{rte^{i\delta}}{1 - Re^{i\delta}}$$

反射光

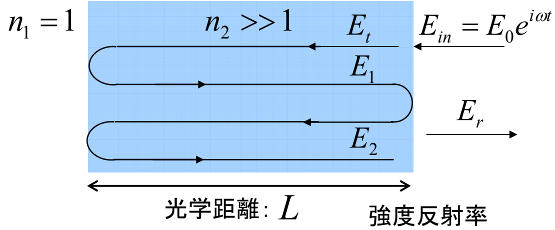
複素反射率

$$E_r = -rE_{in} + tE_{cav} = E_0 \boxed{\frac{r(e^{i\delta} - 1)}{1 - Re^{i\delta}}}$$

自由端反射

$$r = \frac{n_2 - n_1}{n_1 + n_2} \approx 1$$

固定端反射



強度反射率

$$R \equiv r^2$$

$$E_t = tE_0$$

$$E_1 = tE_0 \times re^{i\delta}$$

$$E_2 = tE_0 \times re^{i\delta} \times Re^{i\delta}$$

$$\vdots$$

$$E_n = rte^{i\delta} \times (Re^{i\delta})^{n-1}$$

III. A QUANTITATIVE MODEL

A. Reflection of a monochromatic beam from a Fabry–Perot cavity

To describe the behavior of the reflected beam quantitatively, we can pick a point outside the cavity and measure the electric field over time. The magnitude of the electric field of the incident beam can be written

$$E_{\mathrm{inc}} = E_0 e^{i\omega t}.$$

The electric field of the reflected beam (measured at the same point) is

$$E_{\mathrm{ref}} = E_1 e^{i\omega t}.$$

We account for the relative phase between the two waves by letting E_0 and E_1 be complex. The *reflection coefficient* $F(\omega)$ is the ratio of E_{ref} and E_{inc}, and for a symmetric cavity with no losses it is given by

$$F(\omega) = E_{\mathrm{ref}}/E_{\mathrm{inc}} = \frac{r\left(\exp\left(i\frac{\omega}{\Delta\nu_{\mathrm{fsr}}}\right) - 1\right)}{1 - r^2\exp\left(i\frac{\omega}{\Delta\nu_{\mathrm{fsr}}}\right)}, \quad (3.1)$$

where r is the amplitude reflection coefficient of each mirror, and $\Delta\nu_{\mathrm{fsr}} = c/2L$ is the free spectral range of the cavity of length L.

The beam that reflects from a Fabry–Perot cavity is actually the coherent sum of two different beams: the *promptly reflected beam*, which bounces off the first mirror and never enters the cavity; and a *leakage beam*, which is the small part of the standing wave inside the cavity that leaks back through the first mirror, which is never perfectly reflecting.

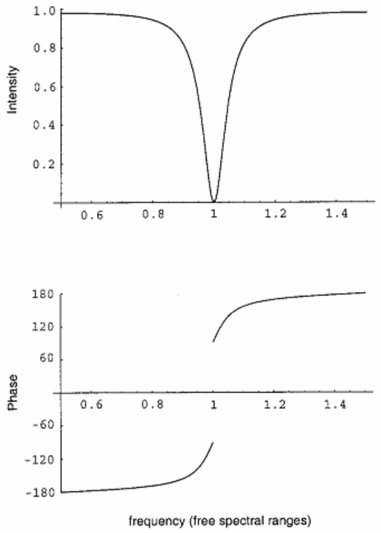


Fig. 4. Magnitude and phase of the reflection coefficient for a Fabry–Perot cavity. As in Fig. 1, the finesse is about 12. Note the discontinuity in phase, caused by the reflected power vanishing at resonance.

Am. J. Phys. **69**, 79 (2001)