

ガウシアンビーム光学 (とGouy位相)

ヘルムホルツ方程式

マクスウェル方程式より、

$$\Delta \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \leftarrow 3\text{次元波動方程式}$$

$$\Leftrightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \begin{pmatrix} E_x(\mathbf{r}, t) \\ E_y(\mathbf{r}, t) \\ E_z(\mathbf{r}, t) \end{pmatrix} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \begin{pmatrix} E_x(\mathbf{r}, t) \\ E_y(\mathbf{r}, t) \\ E_z(\mathbf{r}, t) \end{pmatrix}$$

電磁波は特定の方向にのみ偏光しているとし(近似し)、
その成分を u とする。また時間依存性は $e^{-i\omega t}$ であるとすると

$$\Delta u + k^2 u = 0 \quad \text{ヘルムホルツ方程式}$$
$$(k = \omega/c)$$

ヘルムホルツ方程式の解

$\Delta u + k^2 u = 0$ の解として、 $u = f(x, y, z)e^{ikz}$

の形のものを探してみる。方程式に代入すると、

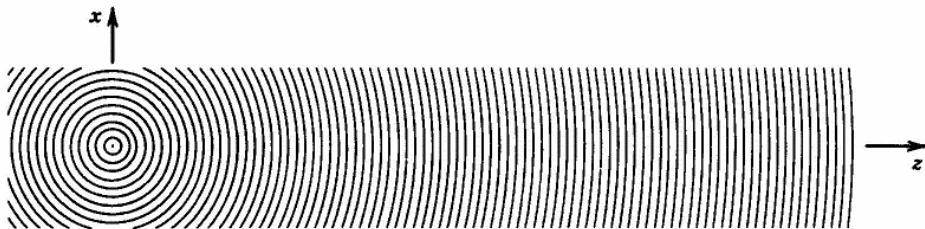
$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} + 2ik \frac{\partial f}{\partial z} = 0$$

波の振幅 f が z 方向に関して(波長スケールで)緩やかに変化する場合 (slowly varying envelope approximation)

$$\frac{\partial^2 f}{\partial z^2} \ll k \frac{\partial f}{\partial z} \rightarrow \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + 2ik \frac{\partial f}{\partial z} = 0$$

Paraxial (近軸)ヘルムホルツ方程式

近軸ヘルムホルツ方程式の解①



Spherical

$$u(\mathbf{r}) = \frac{A}{r} e^{ikr} \xrightarrow{\text{近似}}$$

$$r = (x^2 + y^2 + z^2)^{1/2} \\ = z \left(1 + \frac{x^2 + y^2}{z^2} \right)^{1/2} \\ \cong z + \frac{x^2 + y^2}{2z}$$

Paraboloidal

$$u(\mathbf{r}) = \frac{A}{z} \exp \left[ik \left(z + \frac{x^2 + y^2}{2z} \right) \right] \\ \left(f = \frac{A}{z} \exp \left[ik \frac{x^2 + y^2}{2z} \right] \right)$$

Planar

$$u(\mathbf{r}) = A e^{ikz} \\ (f = 1)$$

この2つは解になっている

近軸ヘルムホルツ方程式の解②

放物面波 (paraboloidal wave) の場合、

$$f = \frac{A}{z} \exp\left[ik \frac{x^2 + y^2}{2z}\right]$$

$z \rightarrow z - iz_0$ と置き換えても、近軸ヘルムホルツ方程式を満たす

$$f = \frac{A}{z - iz_0} \exp\left[ik \frac{x^2 + y^2}{2(z - iz_0)}\right] \quad \leftarrow \text{実はこれが既にガウシアンビームを表している。}$$

関数 $R(z)$, $w(z)$ を次のように定義する:

$$\frac{1}{z - iz_0} = \frac{z + iz_0}{z^2 + z_0^2} = \frac{1}{R(z)} + i \frac{2}{kw^2(z)}$$

$$R(z) \equiv z \left[1 + \left(\frac{z_0}{z} \right)^2 \right] \quad w(z) \equiv w_0 \left[1 + \left(\frac{z}{z_0} \right)^2 \right]^{1/2} \quad w_0 \equiv \left(\frac{2z_0}{k} \right)^{1/2}$$

ガウシアン (Gaussian) ビーム

$$f = \frac{A}{z - iz_0} \exp\left[ik \frac{x^2 + y^2}{2} \left(\frac{1}{R(z)} + i \frac{2}{kw^2(z)} \right)\right]$$

$$= A_0 \frac{w_0}{w(z)} \exp\left[-\frac{\rho^2}{w^2(z)}\right] \exp\left[ik \frac{\rho^2}{2R(z)} - i\zeta(z)\right]$$

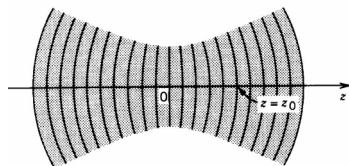
Gouy位相

$$\zeta(z) \equiv \tan^{-1}\left(\frac{z}{z_0}\right)$$

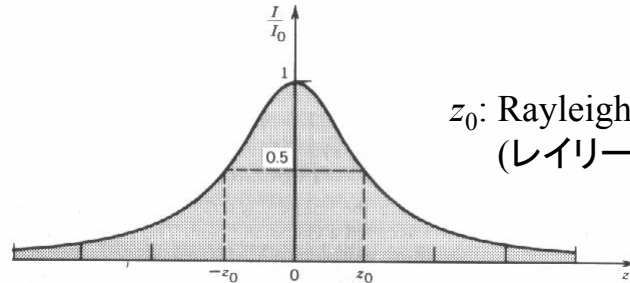
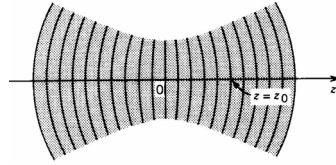
$$\rho \equiv (x^2 + y^2)^{1/2}$$

$$u = fe^{ikz} = A_0 \frac{w_0}{w(z)} \exp\left[-\frac{\rho^2}{w^2(z)}\right] \exp\left[ik \left(z + \frac{\rho^2}{2R(z)} \right) - i\zeta(z)\right]$$

$$I \equiv |u|^2 = I_0 \left[\frac{w_0}{w(z)} \right]^2 \exp\left[-\frac{2\rho^2}{w^2(z)}\right]$$



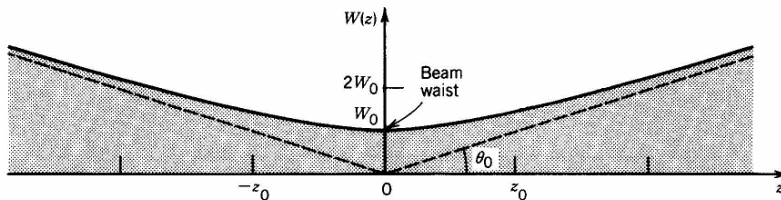
z軸上の強度変化



z_0 : Rayleigh range
(レイリー長)

$$I(0,0,z) = I_0 \left[\frac{w_0}{w(z)} \right]^2 = \frac{I_0}{1 + (z/z_0)^2}$$

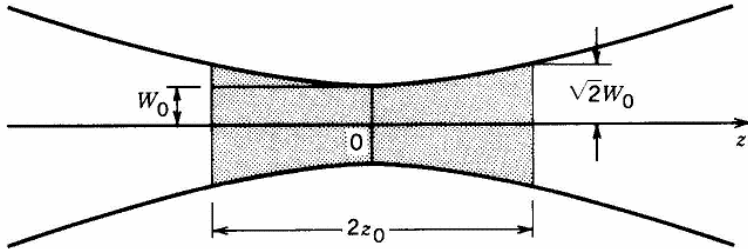
ビーム半径(Beam radius)



$$w(z) \equiv w_0 \left[1 + \left(\frac{z}{z_0} \right)^2 \right]^{1/2} \quad w_0 = \left(\frac{\lambda z_0}{\pi} \right)^{1/2} : \text{beam waist (ビームウエスト)}$$

$$\theta_0 = \lim_{z \rightarrow \infty} \frac{w(z)}{z} = \frac{w_0}{z_0} = \frac{\lambda}{\pi w_0} : \text{divergence angle (広がりの角)}$$

焦点深度 (depth of focus)

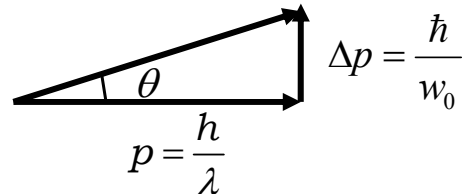
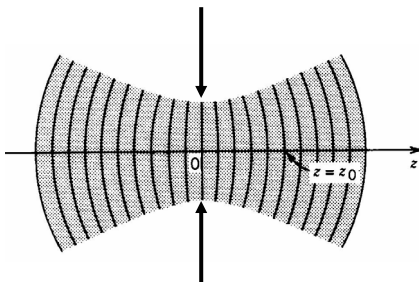


ビーム半径が $\sqrt{2}w_0$ 以下である領域の長さ
(強度が $I_0/2$ 以上である領域の長さ)

$2z_0$: depth of focus (confocal parameter)

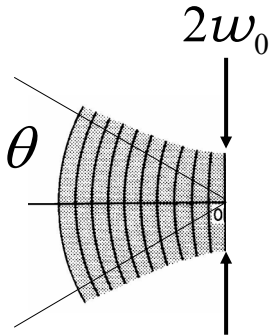
ガウシアンビームと不確定性原理

$$\Delta x = \frac{w_0}{2}, \Delta x \Delta p = \frac{\hbar}{2} \rightarrow \Delta p = \frac{\hbar}{2\Delta x} = \frac{\hbar}{w_0}$$



$$\theta = \frac{2\Delta p}{p} = \frac{2\lambda}{h} \cdot \frac{\hbar}{w_0} = \frac{\lambda}{\pi w_0}$$

ビームはどこまで絞れるのか？



$$\theta \approx \frac{\lambda}{\pi w_0} \rightarrow w_0 \approx \frac{\lambda}{\pi \theta}$$

$$0 < \theta < \frac{\pi}{2} \text{ より } w_0 \approx \lambda$$

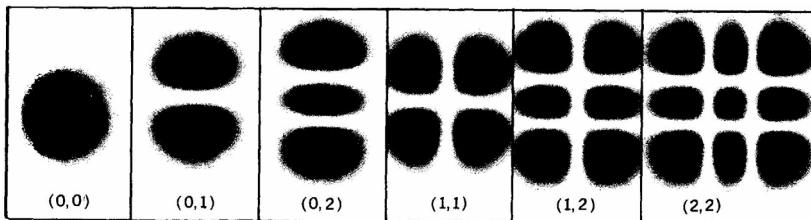
ビームは波長以下には絞れない

エルミート・ガウシアンビーム

$$u_{l,m}(x,y,z) = A_{l,m} \left[\frac{w_0}{w(z)} \right] G_l \left[\frac{\sqrt{2}x}{w(z)} \right] G_m \left[\frac{\sqrt{2}y}{w(z)} \right]$$

$$\times \exp \left[ik \left(z + \frac{x^2 + y^2}{2R(z)} \right) - \underbrace{i(l+m+1)\zeta(z)}_{\text{Gouy位相}} \right]$$

$$\left(G_l(\xi) \equiv H_l(\xi) \exp \left(-\frac{\xi^2}{2} \right) \right)$$



Pulsed polarization gradient cooling in an optical dipole trap with a Laguerre-Gaussian laser beam

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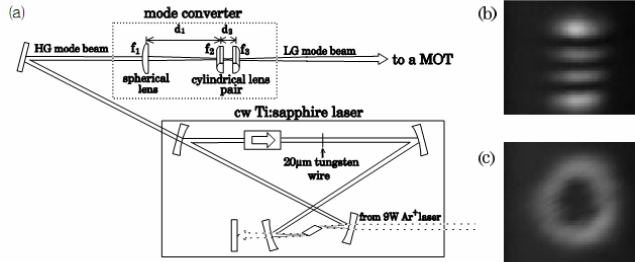


Fig. 1. (a) Experimental setup for generation of Laguerre-Gaussian (LG) beams. Ti:sapphire laser is forced to operate in higher-order Hermite-Gaussian (HG) mode by a 20 μm tungsten wire inside the cavity. The mode converter consists of one spherical lens (focal length: f_1) and two mutually-parallel cylindrical lenses (f_2 and f_3). The axes of the cylindrical lenses make an angle of 45 degrees with the node line of the HG mode beam. Dimensions are: $d_1 = 250$ mm, $d_2 = 35$ mm, $f_1 = 250$ mm, $f_2 = f_3 = 25$ mm. (b) and (c) are the beam intensity profiles monitored before (HG₀₃) and after (LG₀₃) the mode converter, respectively.