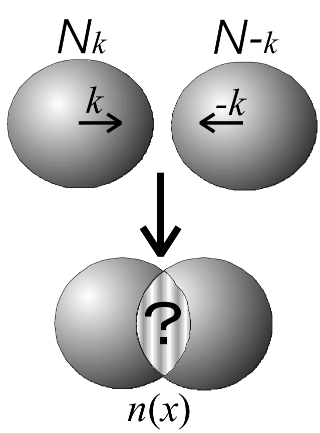
原子の2原子干渉

2010.2.3ランチミーティング
担当：鳥井

独立な2つのBECは干渉するか？



＜粒子数状態の場合＞

$$n(x) = \langle N_k, N_{-k} | \hat{\Psi}^{+}(x)\hat{\Psi}(x) | N_k, N_{-k} \rangle$$
$$= \langle N_k, N_{-k} | \hat{a}_k^{+}\hat{a}_k | N_k, N_{-k} \rangle$$
$$+ \langle N_k, N_{-k} | \hat{a}_{-k}^{+}\hat{a}_{-k} | N_k, N_{-k} \rangle$$
~~$+ \langle N_k, N_{-k} | \hat{a}_k^{+}\hat{a}_{-k} | N_k, N_{-k} \rangle e^{-2ikx}$~~
~~$+ \langle N_k, N_{-k} | \hat{a}_{-k}^{+}\hat{a}_k | N_k, N_{-k} \rangle e^{2ikx}$~~
$$= N_k + N_{-k}(\text{一定})$$ 干渉縞現れない

＜コヒーレント状態の場合＞ $\alpha_k = \sqrt{N_k}e^{i\theta_k}, \alpha_{-k} = \sqrt{N_{-k}}e^{i\theta_{-k}}$

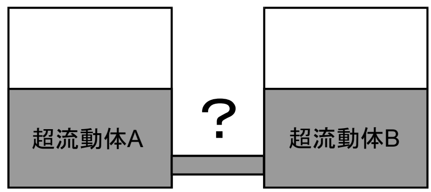
$$n(x) = \langle \alpha_k, \alpha_{-k} | \hat{\Psi}^{+}(x)\hat{\Psi}(x) | \alpha_k, \alpha_{-k} \rangle$$
$$= \langle \alpha_k, \alpha_{-k} | \hat{a}_k^{+}\hat{a}_k | \alpha_k, \alpha_{-k} \rangle$$
$$+ \langle \alpha_k, \alpha_{-k} | \hat{a}_{-k}^{+}\hat{a}_{-k} | \alpha_k, \alpha_{-k} \rangle$$
$$+ \langle \alpha_k, \alpha_{-k} | \hat{a}_k^{+}\hat{a}_{-k} | \alpha_k, \alpha_{-k} \rangle e^{-2ikx}$$
$$+ \langle \alpha_k, \alpha_{-k} | \hat{a}_{-k}^{+}\hat{a}_k | \alpha_k, \alpha_{-k} \rangle e^{2ikx}$$
$$= N_k + N_{-k} + \boxed{2\sqrt{N_k N_{-k}}\cos\{2kx + (\theta_k - \theta_{-k})\}}$$

干渉縞現れる

アンダーソンの思考実験

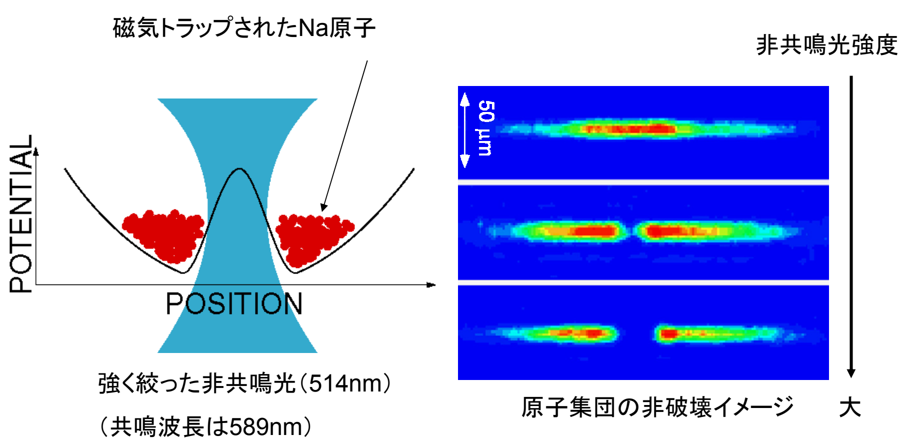
P.W. Anderson, in The Lesson of Quantum,
(Elservier, Amsterdam, 1986) pp. 23-33.

- 離れた2つのバケツ内の超流動体をつなぐと、ジョセフソン流は流れるか？



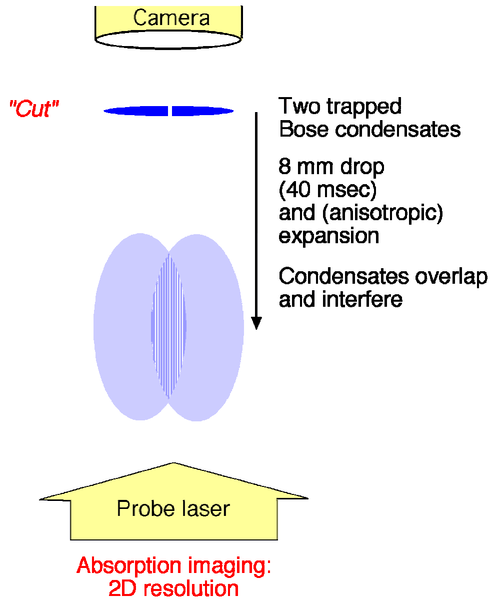
$$\boldsymbol{J} \propto \frac{\hbar}{m}\nabla\phi(\boldsymbol{r})$$

独立なBECの作り方



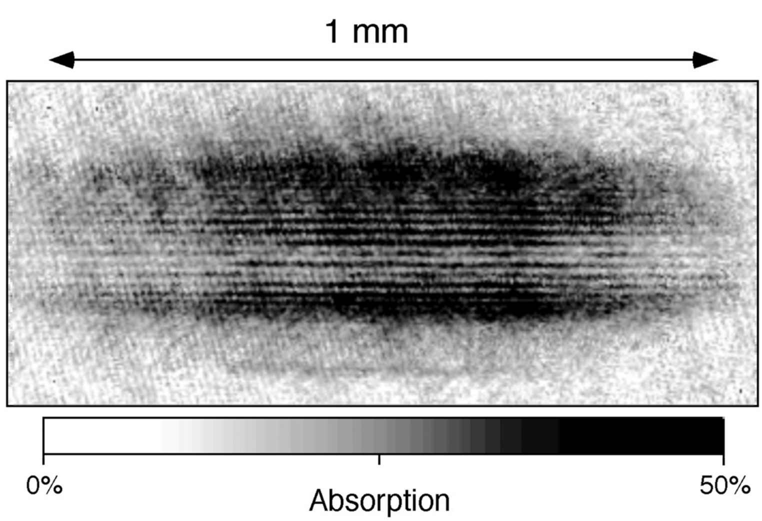
Andrews, Townsend, Miesner, Durfee, Kurn, Ketterle, Science **275**, 589 (1997)

Interference of two condensates



Andrews, Townsend, Miesner, Durfee, Kurn, Ketterle, Science **275**, 589 (1997)

独立なBEC間の干渉



Andrews, Townsend, Miesner, Durfee, Kurn, Ketterle, Science **275**, 589 (1997)

The Feynman Lectures on Physics vol. III Chap.4 (Identical Particle)

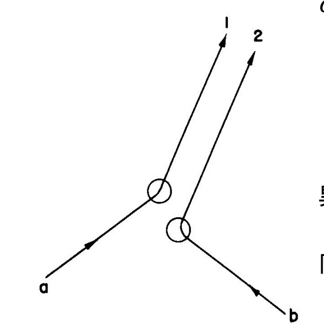


Fig. 4–3. A double scattering into nearby final states.

$a_{1,2}(b_{1,2})$:ボソンa（b）が状態１，2に散乱される確率振幅

粒子a、bが同じ場所で観測される確率は、

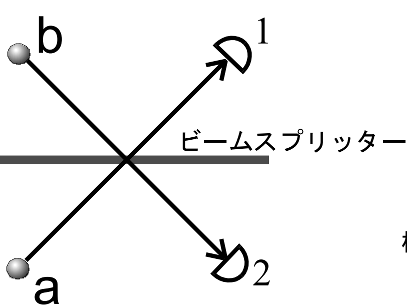
（ $a_1 = a_2 \equiv a$ 、$b_1 = b_2 \equiv b$ ）

異種粒子: $P_2 = |a_1|^2 |b_2|^2 + |a_2|^2 |b_1|^2 = 2|a|^2 |b|^2$

同種粒子: $P_2 = |a_1 b_2 + a_2 b_1|^2 = 4|a|^2 |b|^2$

- もし既にボース粒子がある状態に存在しているなら、次のボース粒子を同じ状態に得る確率は、（既にボース粒子が存在していない場合の）2倍になる

2光子干渉実験



光子a が検出器１で検出される確率振幅: $a_1 = \sqrt{1/2}$

光子a が検出器2で検出される確率振幅: $a_2 = \sqrt{1/2}$

光子b が検出器１で検出される確率振幅: $b_1 = \sqrt{1/2}$

光子b が検出器１で検出される確率振幅: $b_2 = -\sqrt{1/2}$

検出器1，2が同時に光子を検出する確率は、

$$P = |a_1 b_2 + a_2 b_1|^2 = 0$$

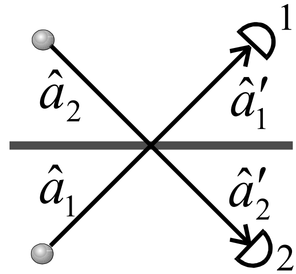
つまり、 $|\text{出力}> = \frac{1}{\sqrt{2}}\{|0_1, 2_2> + |2_1, 0_2>\}$

最初に検出された光子の検出結果によって、次に検出される光子の検出結果が左右される！

演算子（ハイゼンベルグ表示）を使った説明

Input state

$$\hat{a}_1^+ \hat{a}_2^+ |0,0>$$



Beam splitter operation

$$\begin{pmatrix} \hat{a}'_1 \\ \hat{a}'_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

Output state

$$\frac{1}{2}\left(\hat{a}_1'^+ + \hat{a}_2'^+\right)\left(\hat{a}_1'^+ - \hat{a}_2'^+\right)|0,0>$$

$$= \frac{1}{2}\left(\left(\hat{a}_1'^+\right)^2 + \left(\hat{a}_2'^+\right)^2\right)|0,0> = \frac{1}{\sqrt{2}}\left(|2,0> + |0,2>\right)$$

Bunching

VOLUME 59, NUMBER 18 PHYSICAL REVIEW LETTERS 2 NOVEMBER 1987

Measurement of Subpicosecond Time Intervals between Two Photons by Interference

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(Received 10 July 1987)

A fourth-order interference technique has been used to measure the time intervals between two photons, and by implication the length of the photon wave packet, produced in the process of parametric down-conversion. The width of the time-interval distribution, which is largely determined by an interference filter, is found to be about 100 fs, with an accuracy that could, in principle, be less than 1 fs.

PACS numbers: 42.50.Bs, 42.65.Re

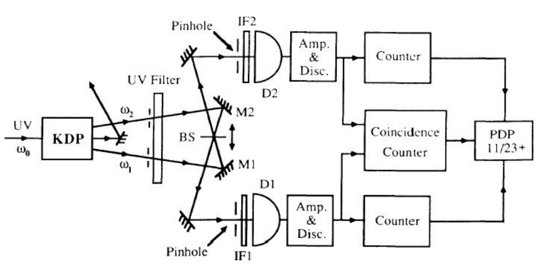


FIG. 1. Outline of the experimental setup.

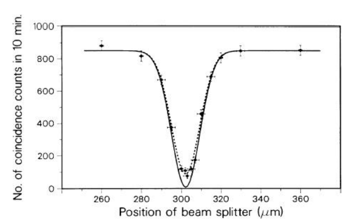
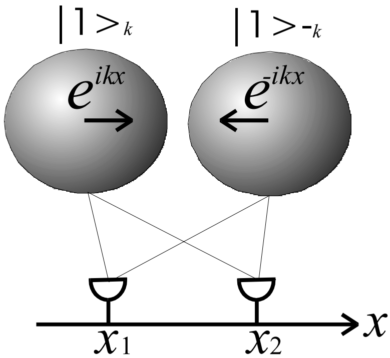


FIG. 2. The measured number of coincidences as a function of beam-splitter displacement $c\,\delta\tau$, superimposed on the solid theoretical curve derived from Eq. (11) with $R/T = 0.95$, $\Delta\omega = 3\times10^{13}$ rad s^{-1}. For the dashed curve the factor $2RT/(R^2+T^2)$ in Eq. (11) was multiplied by 0.9. The vertical error bars correspond to one standard deviation, whereas horizontal error bars are based on estimates of the measurement accuracy.

2つの1原子状態間の干渉

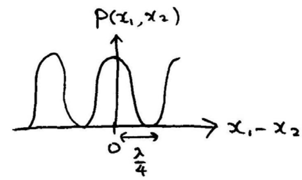


密度演算子の期待値

$$n(x) = < 1_k, 1_{-k} \,|\, \hat{\Psi}^+(x)\hat{\Psi}(x) \,|\, 1_k, 1_{-k} >$$
$$= 2 \ (一定)$$

原子が検出器1と検出器2で検出される確率

$$P(x_1, x_2) \propto \left| e^{ikx_1} \cdot e^{-ikx_2} + e^{ikx_2} \cdot e^{-ikx_1} \right|^2$$
$$= 2\left\{ 1 - \cos 2k(x_1 - x_2) \right\}$$



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Quantum Phase of a Bose-Einstein Condensate with an Arbitrary Number of Atoms

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(Received 18 August 1995)

We study the interference of two Bose-Einstein condensates within an elementary model. The detection of the atoms is modeled by adapting the standard theory of photon detection. Even though the condensates are taken to be in number states with no phases whatsoever, our stochastic simulations of atom detection produce interference patterns as would also be predicted on the basis of the phases of the macroscopic wave functions describing the condensates. In statistical mechanics terms, we have devised a method to analyze spontaneous symmetry breaking for an arbitrary (not necessarily larger) number of particles.

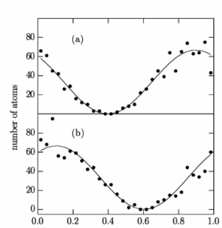

FIG. 1. Numerically simulated histograms (filled circles) for the detected atom positions with $N = 1000$ atoms, for (a) the quantum measurement model and (b) the wave function model. Also shown as solid lines are least-squares fit histograms predicted from the probability distribution of the form $1 + \beta\cos(2\pi x + \varphi)$, with β and φ as the free parameters. In these histograms the positions of the atoms are sorted into $n_b = 30$ equally wide bins.

This work was triggered by a question asked by W. D. Phillips: Are two light beams in number states able to interfere? Incidentally, a straightforward variant of the argument of the present paper shows that the answer is yes. We acknowledge support from the National Science Foundation.