緩和（自然放出）のある場合

$$\frac{dC_2(t)}{dt} = -i\frac{\Omega}{2}C_1(t) - \left(i\frac{\delta}{2}+\gamma\right)C_2(t)$$

$$\frac{d\rho_{22}}{dt} = \frac{dC_2(t)}{dt}C_2^*(t) + C_2(t)\frac{dC_2^*(t)}{dt}$$

$$= -2\gamma\rho_{22} + i\frac{\Omega}{2}(\tilde{\rho}_{21} - \tilde{\rho}_{12})$$

$$= -\Gamma\rho_{22} - \Omega\,\mathrm{Im}(\tilde{\rho}_{21})$$

（$\Gamma \equiv 2\gamma$：自然放出レート）

$$\frac{d\tilde{\rho}_{12}}{dt} = \frac{dC_1(t)}{dt}C_2^*(t) + C_1(t)\frac{dC_2^*(t)}{dt}$$

$$= (-i\delta - \gamma)\tilde{\rho}_{12} - i\frac{\Omega}{2}(\rho_{22} - \rho_{11})$$

$$\frac{d\rho_{11}}{dt} = \frac{dC_1(t)}{dt}C_1^*(t) + C_1(t)\frac{dC_1^*(t)}{dt}$$

$$= i\frac{\Omega}{2}(\tilde{\rho}_{12} - \tilde{\rho}_{21})$$

確率保存しない

$$\rho_{11} + \rho_{22} \neq 1$$

⬇

$$\frac{d\rho_{11}}{dt} = \Gamma\rho_{22} + i\frac{\Omega}{2}(\tilde{\rho}_{12} - \tilde{\rho}_{21})$$

$$= \Gamma(1 - \rho_{11}) + \Omega\,\mathrm{Im}(\tilde{\rho}_{21}).$$

ブロッホベクトル

$$U \equiv \tilde{\rho}_{12} + \tilde{\rho}_{21} = 2\,\mathrm{Re}(\tilde{\rho}_{21}) \longleftrightarrow \chi'$$

$$V \equiv i(\tilde{\rho}_{12} - \tilde{\rho}_{21}) = 2\,\mathrm{Im}(\tilde{\rho}_{21}) \longleftrightarrow \chi''$$

$$W \equiv \rho_{22} - \rho_{11}$$

$$\begin{cases} \dfrac{dW}{dt} = -\Gamma(W+1) - \Omega V \\ \dfrac{dU}{dt} = -\gamma U - \delta V \\ \dfrac{dV}{dt} = -\gamma V + \delta U + \Omega W \end{cases}$$

$$\xrightarrow{\gamma = 0}$$

$$\begin{cases} \dfrac{d\vec{\rho}}{dt} = \vec{\Omega} \times \vec{\rho} \\ \vec{\Omega} \equiv (-\Omega, 0, \delta) \\ \left|\vec{\Omega}\right| = \sqrt{\Omega^2 + \delta^2} = \Omega' \end{cases}$$

JOURNAL OF APPLIED PHYSICS VOLUME 28, NUMBER 1 JANUARY, 1957

Geometrical Representation of the Schrödinger Equation for Solving Maser Problems

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(Received September 18, 1956)

A simple, rigorous geometrical representation for the Schrödinger equation is developed to describe the behavior of an ensemble of two quantum-level, noninteracting systems which are under the influence of a perturbation. In this case the Schrödinger equation may be written, after a suitable transformation, in the form of the real three-dimensional vector equation $d\mathbf{r}/dt=\boldsymbol{\omega}\times\mathbf{r}$, where the components of the vector $\mathbf{r}$ uniquely determine ψ of a given system and the components of $\boldsymbol{\omega}$ represent the perturbation. When magnetic interaction with a spin $\frac{1}{2}$ system is under consideration, "r" space reduces to physical space. By analogy the techniques developed for analyzing the magnetic resonance precession model can be adapted for use in any two-level problems. The quantum-mechanical behavior of the state of a system under various different conditions is easily visualized by simply observing how $\mathbf{r}$ varies under the action of different types of $\boldsymbol{\omega}$. Such a picture can be used to advantage in analyzing various MASER-type devices such as amplifiers and oscillators. In the two illustrative examples given (the beam-type MASER and radiation damping) the application of the picture in determining the effect of the perturbing field on the molecules is shown and its interpretation for use in the complex Maxwell's equations to determine the reaction of the molecules back on the field is given.

$$\psi(t)=a(t)\psi_a+b(t)\psi_b \quad (1)$$

$$d\mathbf{r}/dt=\boldsymbol{\omega}\times\mathbf{r} \quad (4)$$

$$\begin{aligned} r_1&\equiv ab^*+ba^*\\ r_2&\equiv i(ab^*-ba^*)\\ r_3&\equiv aa^*-bb^*. \end{aligned} \quad (2)$$

$$\begin{aligned} \omega_1&\equiv (V_{ab}+V_{ba})/\hbar\\ \omega_2&\equiv i(V_{ab}-V_{ba})/\hbar\\ \omega_3&\equiv \omega_0. \end{aligned} \quad (5)$$

定常状態の解

$$U = \frac{2\delta}{\Omega}\frac{s(\delta)}{1+s(\delta)},\ V = -\frac{2\gamma}{\Omega}\frac{s(\delta)}{1+s(\delta)},\ W = -\frac{1}{1+s(\delta)}$$

$$\left(s(\delta) \equiv s_0 L(\delta), \quad s_0 \equiv \frac{\Omega^2}{2\gamma^2}, \quad L(\delta) \equiv \frac{1}{1+(\delta/\gamma)^2} \right)$$

$$\tilde{\rho}_{21} = \frac{1}{2}(U+iV) = \frac{s(\delta)}{1+s(\delta)} \cdot \frac{\delta - i\gamma}{\Omega}$$

$$\chi = \frac{2d_{12}\tilde{\rho}_{21}}{\varepsilon_0 E_0} = \frac{d_{12}^2}{\varepsilon_0 \hbar \gamma^2} \cdot \frac{L(\delta)}{1+s(\delta)}(-\delta + i\gamma)$$

自然放出レートと双極子モーメントとの関係

ウィグナー･ワイスコップの自然放出の理論 (1930)**より**

$$\Gamma \equiv 2\gamma = \frac{d_{12}^2 \omega^3}{3\pi\varepsilon_0 \hbar c^3} = \frac{d_{12}^2 k^3}{3\pi\varepsilon_0 \hbar} \rightarrow d_{12}^2 = \frac{6\pi\varepsilon_0 \hbar\gamma}{k^3}$$

したがって、

$$\chi = 6\pi\lambdabar^3 \frac{L(\delta)}{1+s(\delta)} \cdot \frac{-\delta + i\gamma}{\gamma} \quad \left(\lambdabar \equiv \frac{\lambda}{2\pi} \right)$$

$$s_0 \equiv \frac{\Omega^2}{2\gamma^2} = \frac{d_{12}^2 E_0^2}{2\hbar^2\gamma^2} = \frac{1}{2}\varepsilon_0 c E_0^2 \cdot \frac{6\pi\lambdabar^3}{c\hbar\gamma} = I / I_s \quad \left(I_s \equiv \frac{c\hbar\gamma}{6\pi\lambdabar^3} \right)$$

飽和強度

2準位原子の複素電気感受率

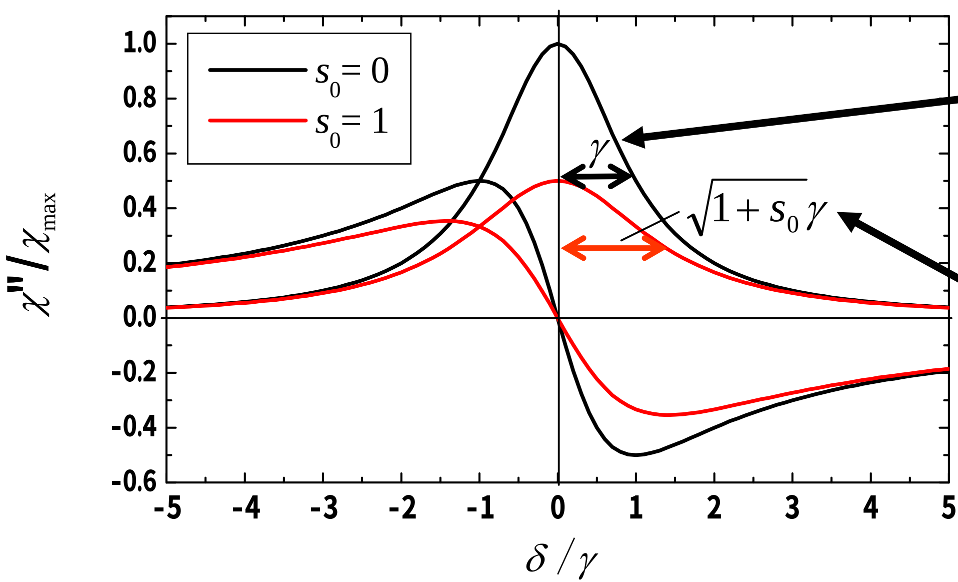
量子論的2準位原子

$$
\begin{cases}
\chi' = -6\pi\bar{\lambda}^3\gamma\dfrac{\delta}{\delta^2+(1+s_0)\gamma^2} \\
\chi'' = 6\pi\bar{\lambda}^3\gamma\dfrac{\gamma}{\delta^2+(1+s_0)\gamma^2}
\end{cases}
$$

$$\gamma \to \sqrt{1+s_0}\,\gamma$$

古典的調和振動子

$$
\begin{cases}
\chi' = -A\dfrac{\delta}{\delta^2+\gamma^2} \\
\chi'' = A\dfrac{\gamma}{\delta^2+\gamma^2}
\end{cases}
$$



自然幅
(natural linewidth)

飽和(パワー)広がり
(power broadening)

$$\sqrt{1+s_0}\,\gamma = \sqrt{1+I/I_s}\,\gamma$$

原子の吸収断面積

密度nの原子気体の吸収係数

$$\alpha = nk\chi'' = n\sigma(\delta)$$

↑

原子数密度

$$\sigma(\delta) \equiv \sigma_0 \frac{\gamma^2}{\delta^2 + (1+s_0)\gamma^2}$$ ：吸収断面積

$$\sigma_0 \equiv 6\pi\lambda^2$$ ：共鳴吸収断面積

$$I(z) = I_0 e^{-\alpha z} = I_0 e^{-n\sigma(\delta)z}$$ ⟵ 光学密度 (optical density)

ここまでのまとめ（要点）

1. 2準位原子の複素電気感受率は、飽和パラメタ $s_0 = I/I_s << 1$ では古典的調和振動子と同じである。

2. 2準位原子の場合、共鳴の幅 Γ を決めるのは、自然放出（励起準位の寿命）である($\Gamma = 1/\tau$)。

3. $s_0 << 1$ でない場合、幅は $(1+s_0)^{1/2}$ 倍に広がり、ピークは $1/(1+s_0)$ 倍に下がる（飽和広がり）。

4. 1原子あたりの共鳴吸収断面積は、遷移モーメントによらず $\sigma_0 = 6\pi\bar{\lambda}^2$。