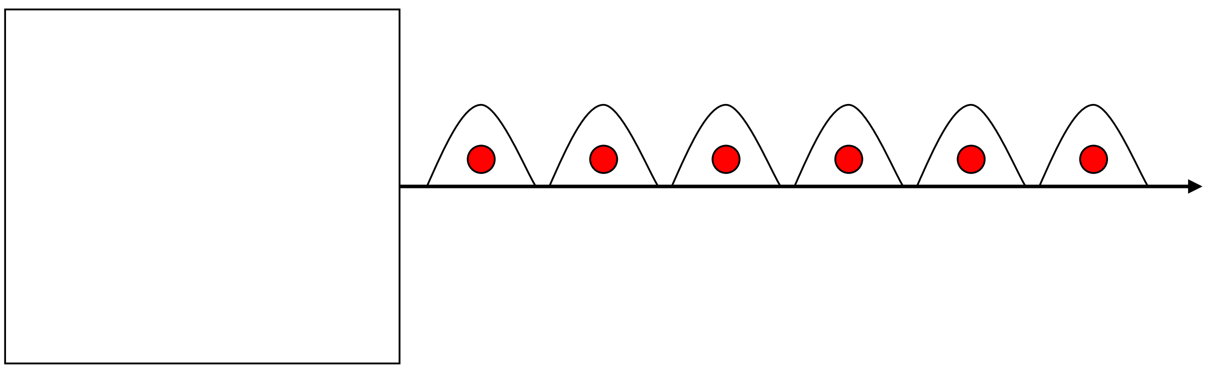
# 21世紀の光源：単一光子発生器

基礎科学科談話会　7月8日

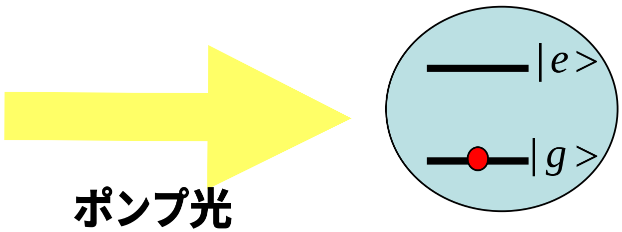
総合文化研究科
鳥井　寿夫

# 理想的な単一光子発生器



常に1光子を含む光パルスを、同一のモード
に発生させる装置

# 励起された原子からの発光



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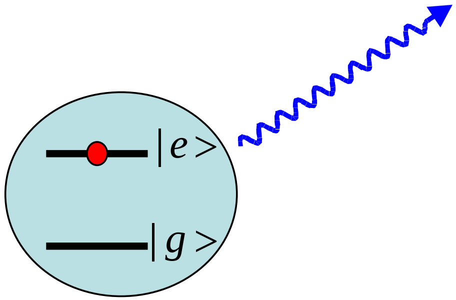

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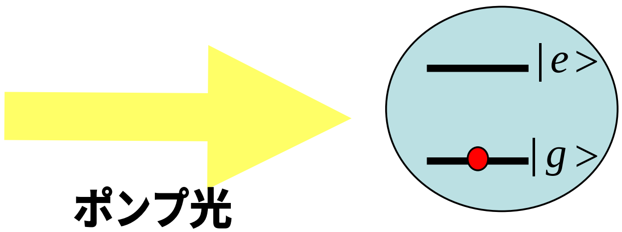

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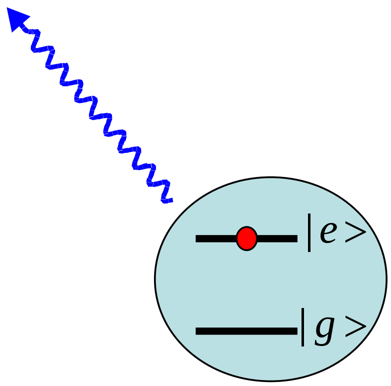

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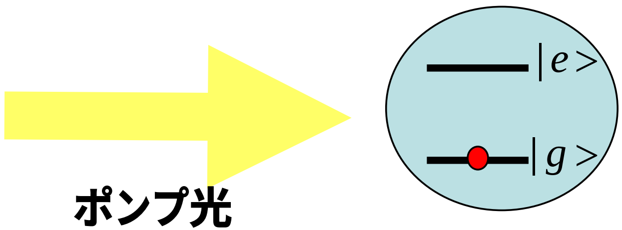
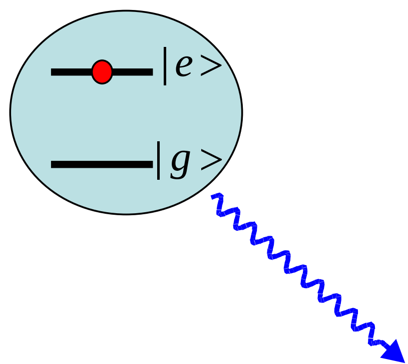


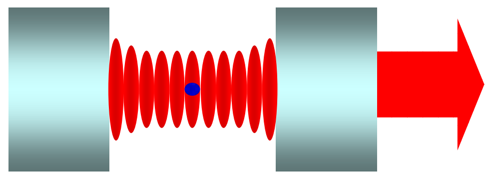
# 励起された原子からの発光



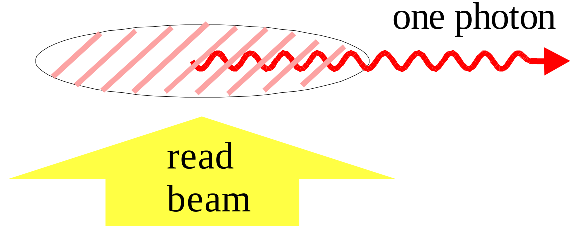

自由空間に孤立した励起原子は、デタラメな方向
(正確にはダイポールの放射パターン) に光子を吐く

# 解決法

- 原子の周りに共振器を置いて、特定のモードに光子を放出させる（パーセル効果）

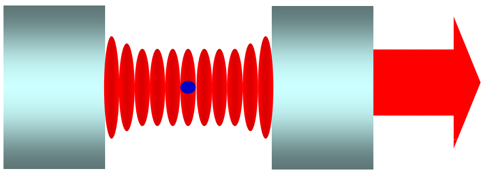


- 原子集団のコヒーレンスを用いて、特定の方向に光子を放出させる（協調効果）



# パーセル因子 (自然放出レートとの比)

**励起された原子が、共振器の特定のモードに光子を吐くレート** (Fermi's golden rule)



$$R = \frac{2\pi}{\hbar^2} \left| < g,1 | \hbar g_0 (a\sigma^+ + a^+\sigma) | e,0 > \right|^2 \delta(\omega_C - \omega_A)$$

$$= 2\pi g_0^2 \frac{\kappa/\pi}{\kappa^2 + \delta^2} \xrightarrow{\omega_c = \omega_A} \frac{2g_o^2}{\kappa}$$

**共振器のモード密度**

$$\left( g_0 \equiv \sqrt{\frac{d_{eg}^2 \omega_C}{2\varepsilon_0 \hbar V}}, \quad d_{eg} \equiv < e | -e\hat{x} | g >, \quad 2\kappa = \frac{1}{\tau_c} = \frac{\pi c}{lF}, \quad V = \frac{\pi}{4} w_0^2 \cdot l \right)$$

**共振器に吐くレートRと自然放出レート Γとの比 (パーセル因子 )**

$$\frac{R}{\Gamma} = \frac{2g_0^2}{\kappa\Gamma} = 2C = \frac{3\lambda^3}{4\pi^2}\left(\frac{Q}{V}\right) \qquad \left( Q \equiv \frac{\omega}{\Delta\omega} = \frac{2l}{\lambda} F \right)$$

Single-atom Cooperativity parameter:

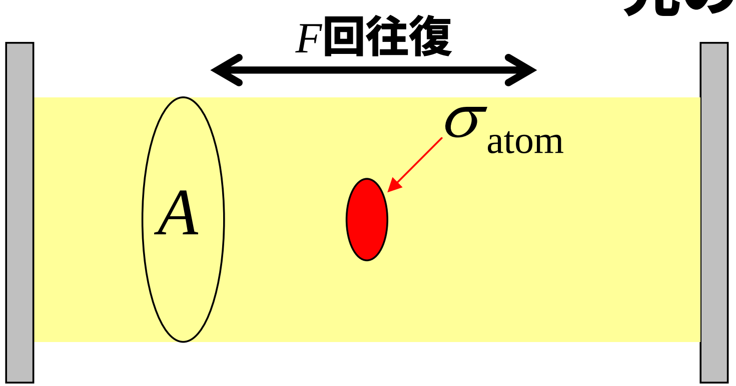
$$C \equiv \frac{g_0^2}{\kappa\Gamma} = \frac{12F}{\pi w_0^2 k^2} = \frac{F}{2\pi} \frac{\sigma_{\text{atom}}}{A} \qquad \left( \sigma_{\text{atom}} = 6\pi\bar{\lambda}^2, A = \frac{\pi}{4} w_0^2 \right)$$

# Cooperatively parameterの意味

光の往復回数　　散乱断面積

$$C \equiv \frac{g_0^2}{\kappa\Gamma} = \frac{12F}{\pi w_0^2 k^2} = \frac{F}{2\pi}\frac{\sigma_{\mathrm{atom}}}{A}$$

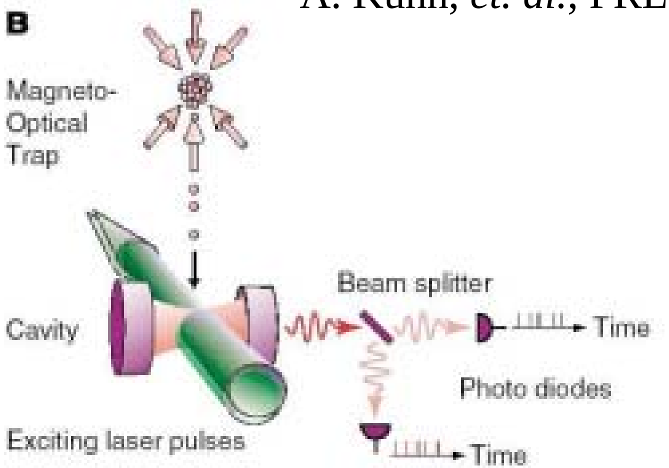
光の断面積

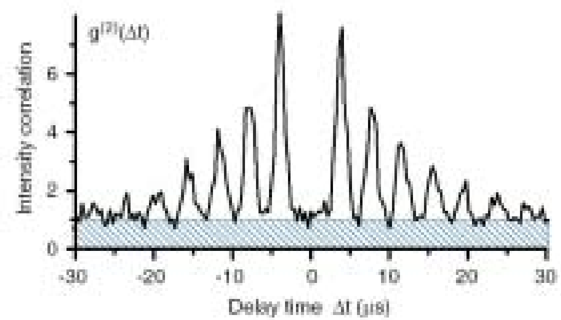


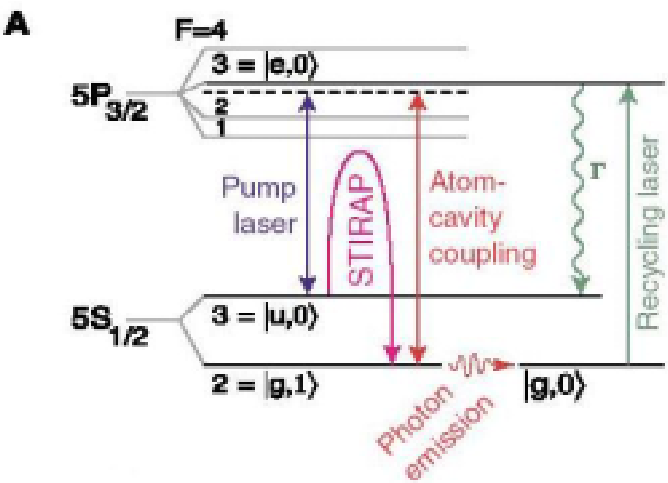

共振器内の光子が原子に吸収される確率

# 微小共振器を用いた単一光子発生器

A. Kuhn, *et. al.*, PRL **89**, 067901 (2002)



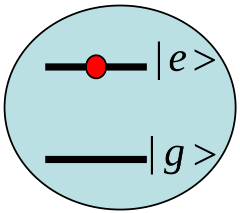




$$|\psi>_{\mathrm{Dark}}=\frac{1}{\sqrt{\Omega_{\mathrm{pump}}^{2}+\Omega_{\mathrm{cavity}}^{2}}}\left(\Omega_{\mathrm{cavity}}\,|u,0>-\Omega_{\mathrm{pump}}\,|g,1>\right)$$

Stimulated Raman Adiabatic Passage

# 1原子の自然放出

## 電気双極子演算子




$$\hat{d} = d(\sigma^{+} + \sigma^{-}) \quad \left(\sigma^{+} = |e><g|, \sigma^{-} = |g><e|\right)$$

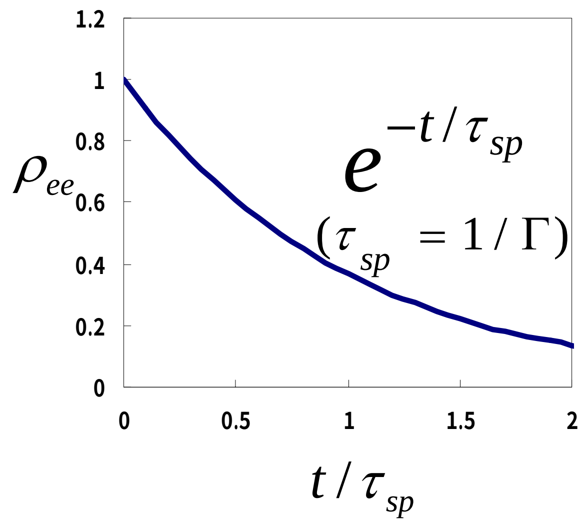
## 原子と電磁場との電気双極子相互作用

$$\hat{H}_{\text{int}} = -\hat{d} \cdot \hat{E}_{rad}$$

## 1原子の自然放出レート

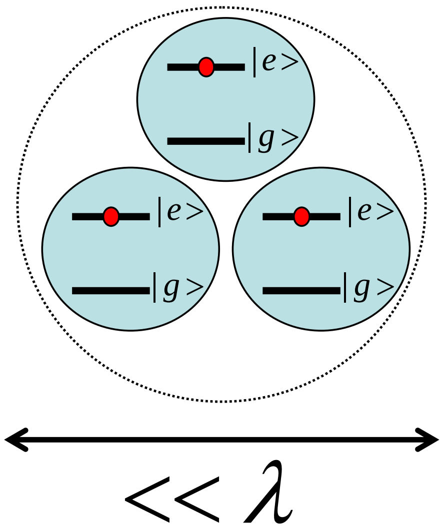
$$R = \Gamma \langle \sigma^{+} \sigma^{-} \rangle \quad \left( \Gamma = \frac{d^{2} \omega^{3}}{3\pi \hbar \varepsilon_{0} c^{3}} \right)$$

励起状態の存在確率




Wigner and Weisskopf (1930)

# 3原子の自然放出




**原子集団の電気双極子演算子**

$$\hat{d} = d\sum_{i=1}^{3}(\sigma_i^+ + \sigma_i^-) \equiv d(\sigma^+ + \sigma^-)$$

$$\left( \sigma^+ \equiv \sum_{i=1}^{3} |e>_{i\,i}< g|, \sigma^+ \equiv \sum_{i=1}^{3} |g>_{i\,i}< e| \right)$$

**自然放出レート**

$$R = \Gamma\left\langle \sigma^+ \sigma^- \right\rangle$$

**1原子の場合と同じように見えるが…**

# Evolution of the three-atom system

$$|e,e,e>$$

$$R=\Gamma\left\langle\sigma^{+}\sigma^{-}\right\rangle=3\Gamma$$

$$\frac{1}{\sqrt{3}}\left(|g,e,e>+|e,g,e>+|e,e,g>\right)$$

$$R=\Gamma\left\langle\sigma^{+}\sigma^{-}\right\rangle=4\Gamma$$

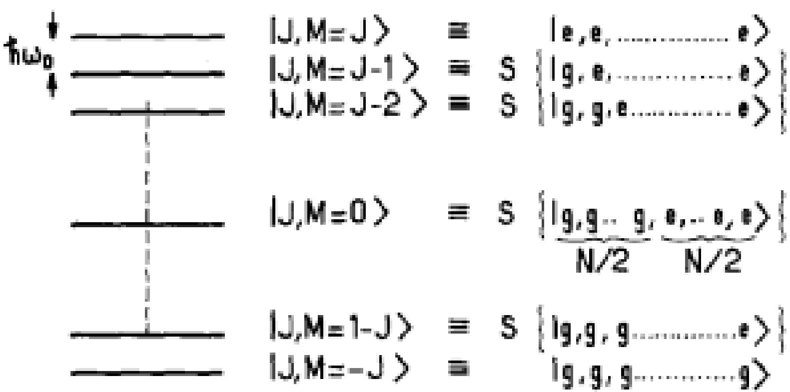
$$\frac{1}{\sqrt{3}}\left(|e,g,g>+|g,e,g>+|g,g,e>\right)$$

$$R=\Gamma\left\langle\sigma^{+}\sigma^{-}\right\rangle=3\Gamma$$

$$|g,g,g>$$

# N原子の自然放出

N-**原子系** ⇔ N spin-1/2 system with the total spin J = N/2
(**必要条件**: **光子の放出に関してN原子が区別がつかない**)




R. H. Dicke, Phys. Rev. **93**, 99 (1954)

Spontaneous emission rate of the N-atom system:

$$\Gamma_N = \Gamma \langle J, M \,|\, J_+ J_- \,|\, J, M \rangle$$
$$= \Gamma (J + M)(J - M + 1)$$
$$= \Gamma N_e (N_g + 1)$$

Enhancement by the number of photons already emitted

# Comparison between ordinary and superradiant emission

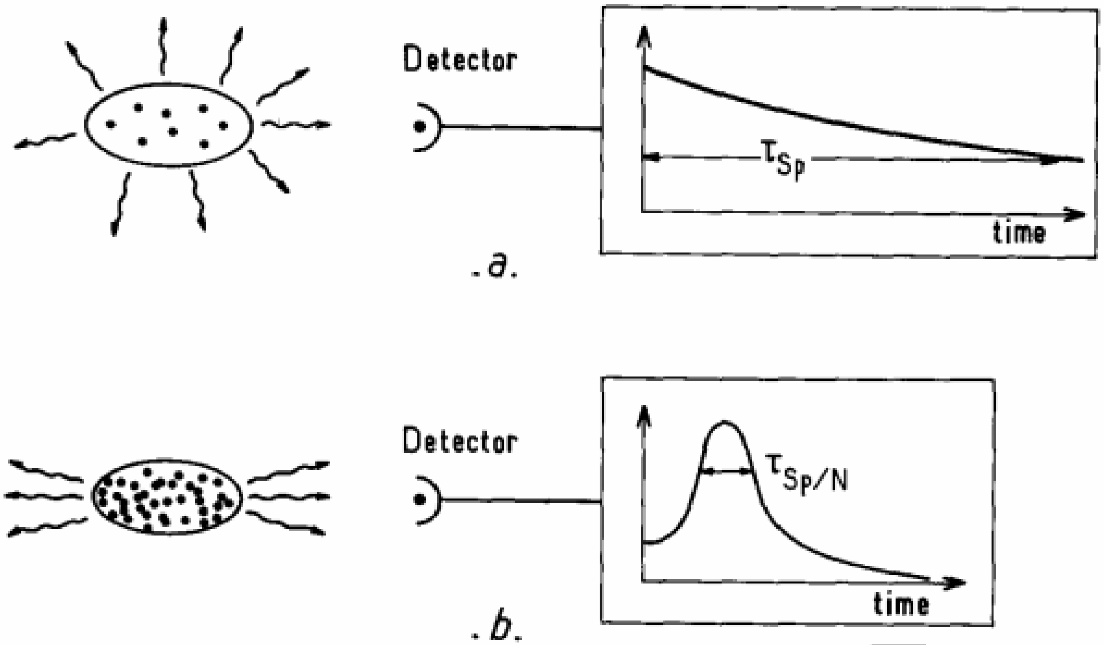


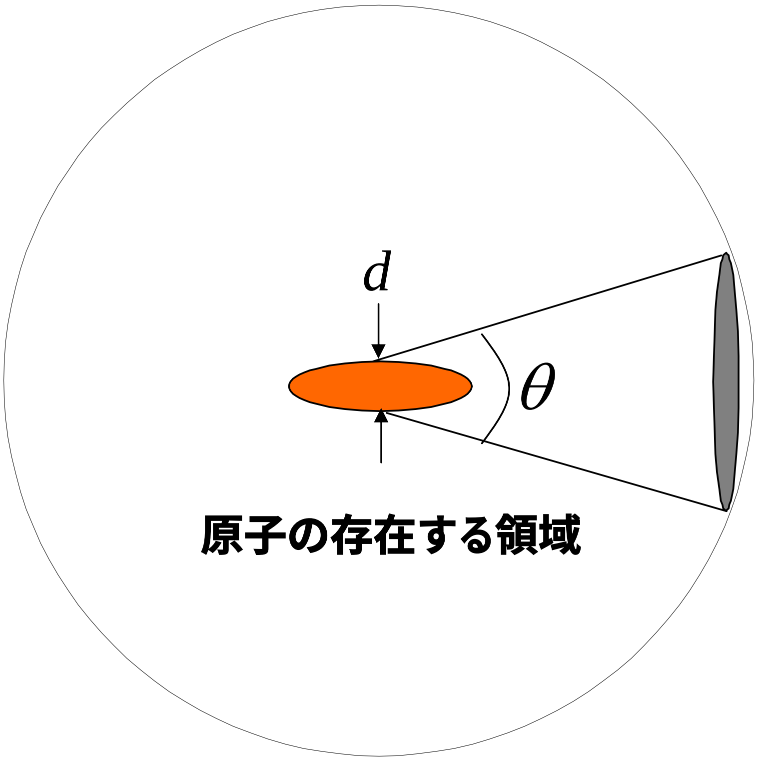

Fig. 1. Comparison between the general characteristics of ordinary fluorescence and superradiance experiments. (a) Ordinary spontaneous emission is essentially isotropic with an exponentially decaying intensity (time constant $\tau_{sp}$). (b) Superradiance is anisotropic with an emission occurring in a short burst of duration $\sim\tau_{sp}/N$.

From M. Gross and S. Haroche, Phys. Rep. **93**, 301 (1982)

# Cooperatively parameterの
# もうひとつの意味

**自由空間** (F=1) **の場合**

$$C = \frac{\sigma_{\mathrm{atom}}}{A} \approx \left(\frac{\lambda}{d}\right)^2$$




**直径dの光源からコヒーレントに出てくる光の回折角**

$$\theta \approx \frac{\lambda}{d}$$

**直径dの光源からコヒーレントに出てくる光の立体角**

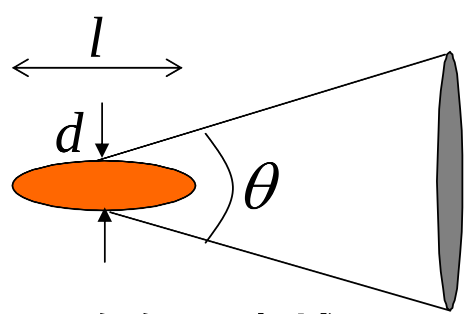
$$\Omega \approx \left(\frac{\lambda}{d}\right)^2 \approx C$$

# Cooperatively parameterの
# 更に別の意味（光学密度）

自由空間（F=1）かつN原子の場合

$$C \approx N\frac{\sigma_{\text{atom}}}{A} = \frac{N}{Al}\sigma_{\text{atom}}l = n\sigma_{\text{atom}}l$$

光学密度




N原子の存在する領域

光学密度の高い原子集団（BEC）のCは1を超える！

# The properties of BEC

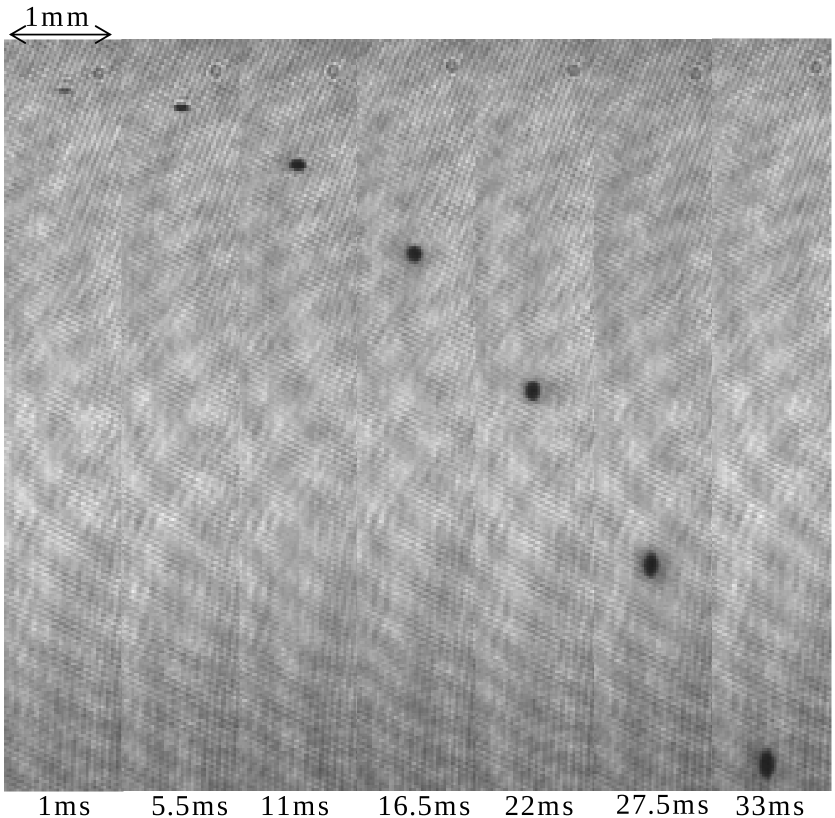
1. **narrow velocity width below the recoil velocity （6 mm/s for Rb87 ）**

2. **Well localized in space （10 μm～100 μm）**

3. **Spatial density of ～$10^{14}$ atoms/$cm^3$**

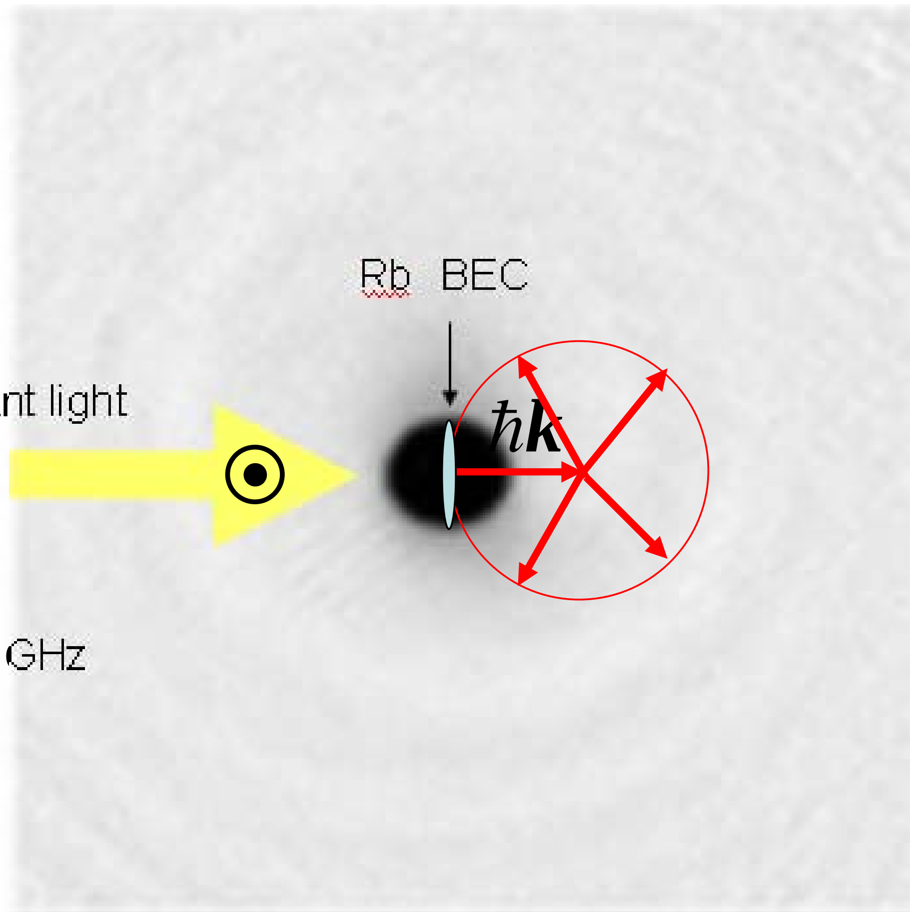
4. **Coherent**

**Images of a BEC released from the magnetic trap**



# Rayleigh scattering in a Rb BEC

Off-resonant light




63 mW/cm$^2$

detuning: -4.4 GHz

Pulse duration:
25 μs-3200 μs

30 ms TOF

D. Schneble, Y.T., M. Boyd, E. W. Streed, D. E. Pritchard, and W. Ketterle, Science **300**, 475 (2003)

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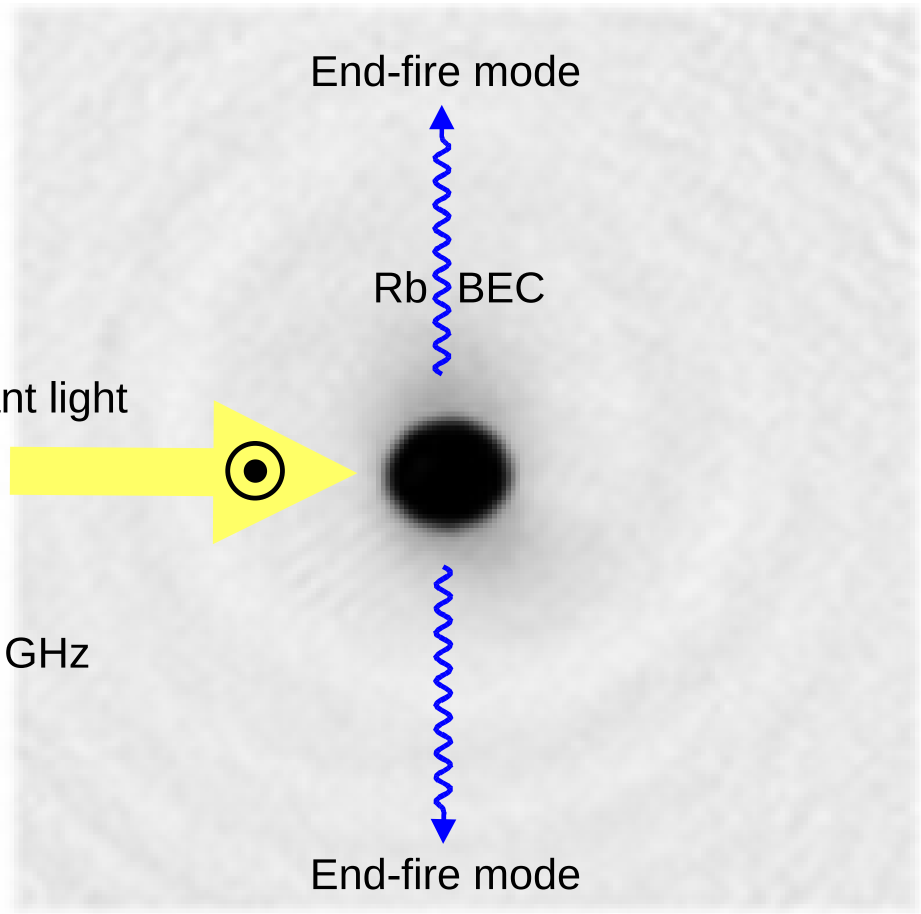


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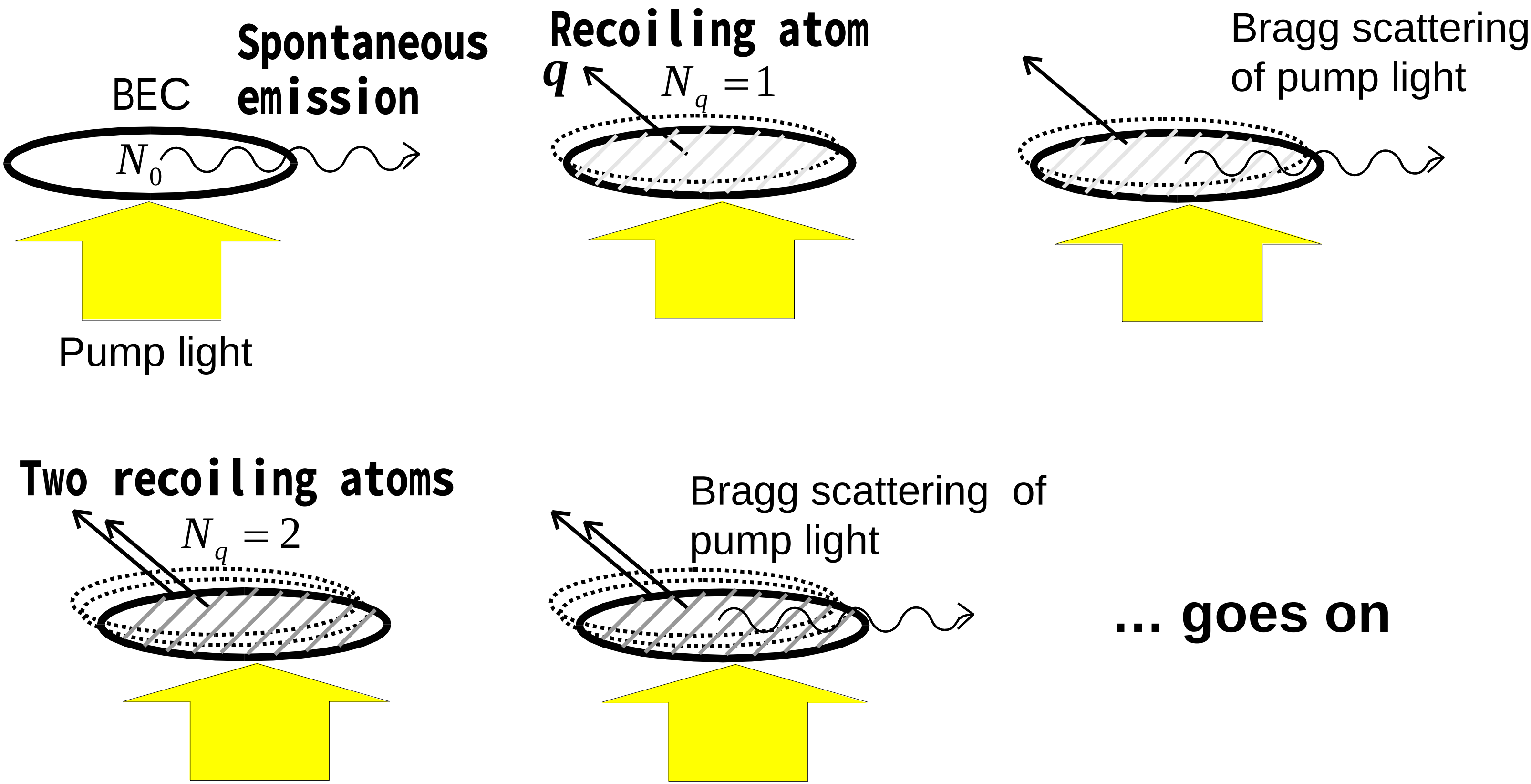
# Semi-classical interpretation of superradiance

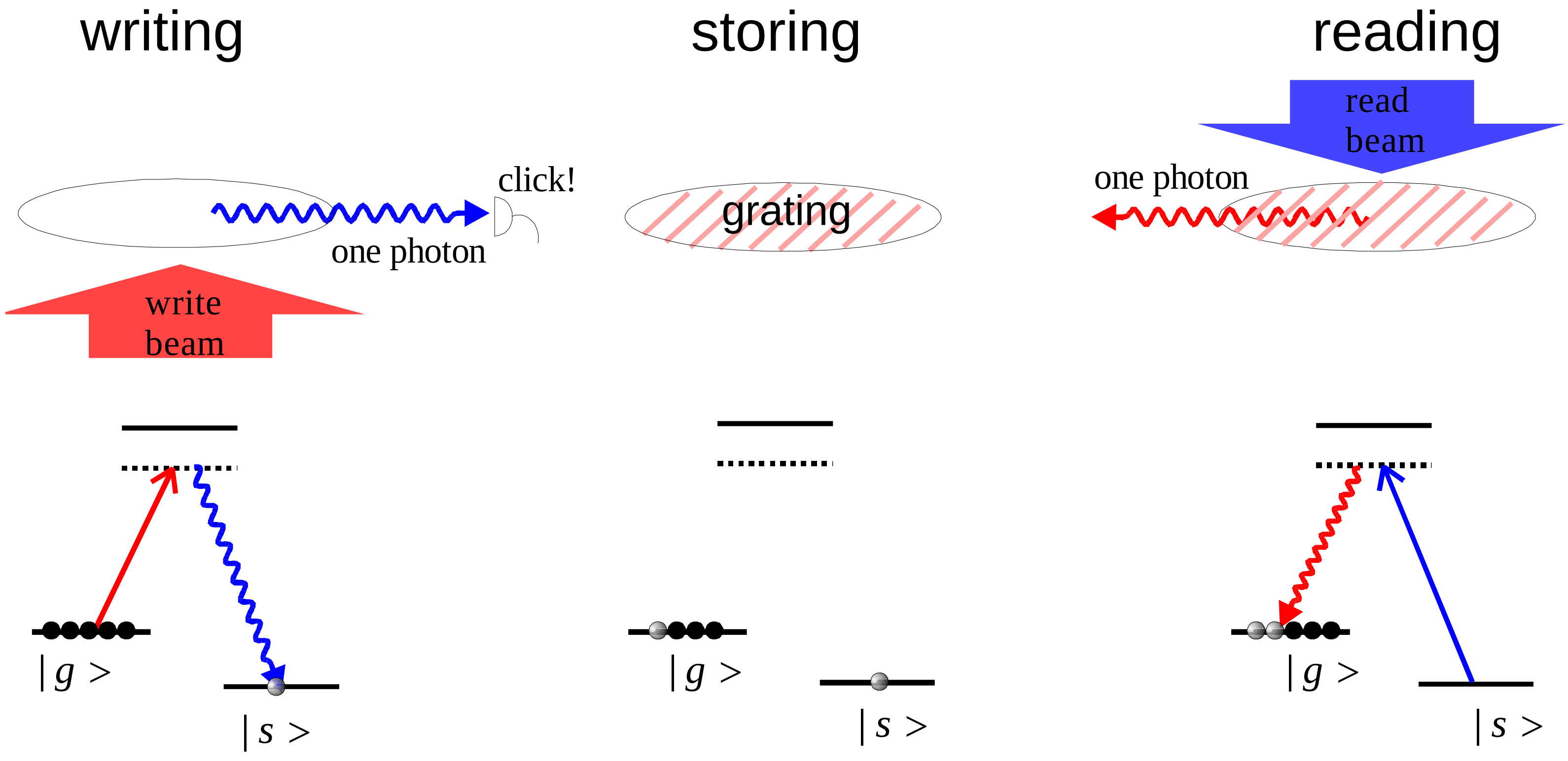



**The rate of light scattering is enhanced by the number of recoiling atoms**

$$\dot{N}_q \propto N_0 (N_q + 1)$$

**Lasing of matter-wave**

# Writing, storing, and reading of a single photon

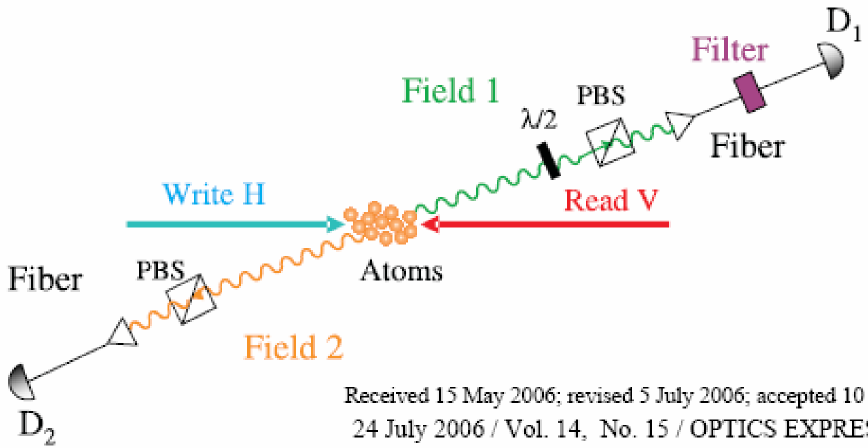


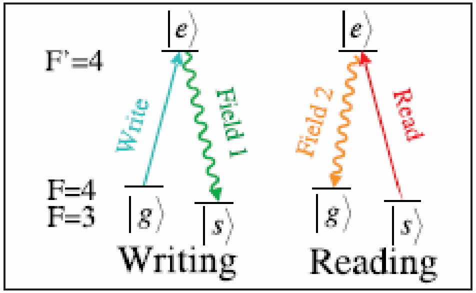
# Efficient retrieval of a single excitation stored in an atomic ensemble

**Julien Laurat, Hugues de Riedmatten, Daniel Felinto, Chin-Wen Chou, Erik W. Schomburg, and H. Jeff Kimble**

*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125, USA*

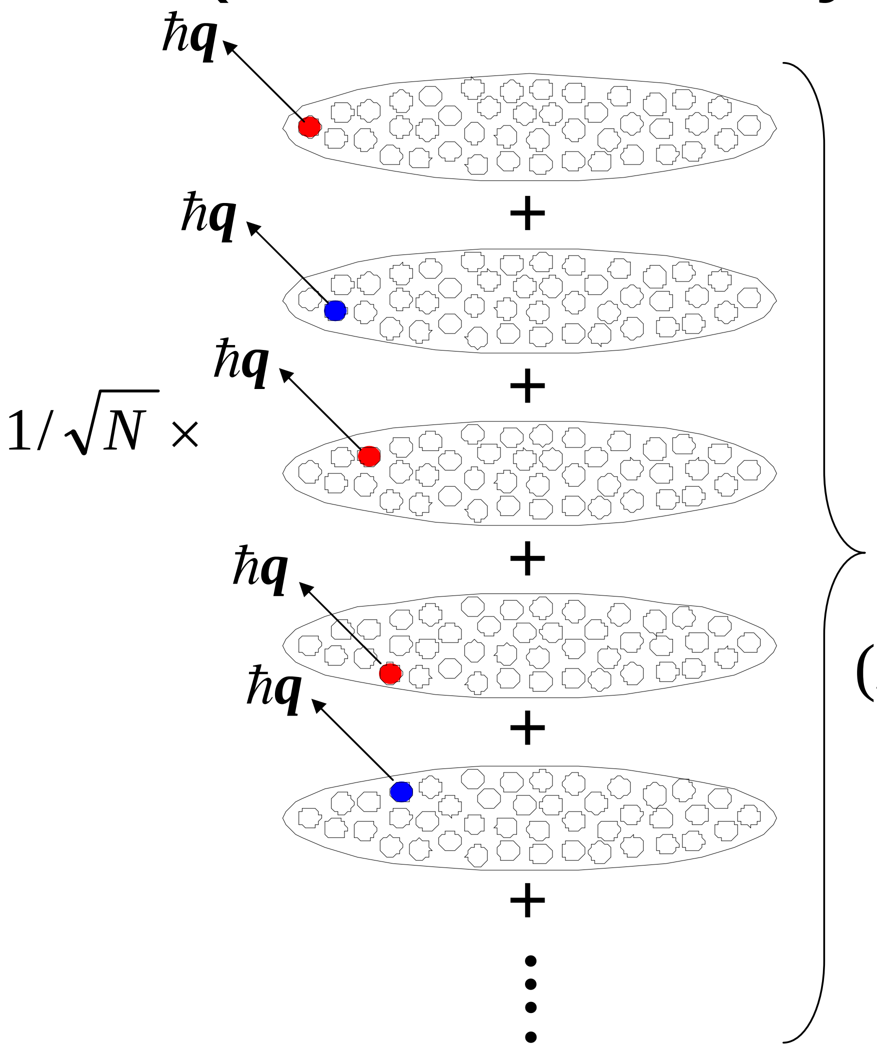
**Abstract:** We report significant improvements in the retrieval efficiency of a single excitation stored in an atomic ensemble and in the subsequent generation of strongly correlated pairs of photons. A 50% probability of transforming the stored excitation into one photon in a well-defined spatio-temporal mode at the output of the ensemble is demonstrated. These improvements are illustrated by the generation of high-quality heralded single photons with a suppression of the two-photon component below 1% of the value for a coherent state. A broad characterization of our system is performed for different parameters in order to provide input for the future design of realistic quantum networks.



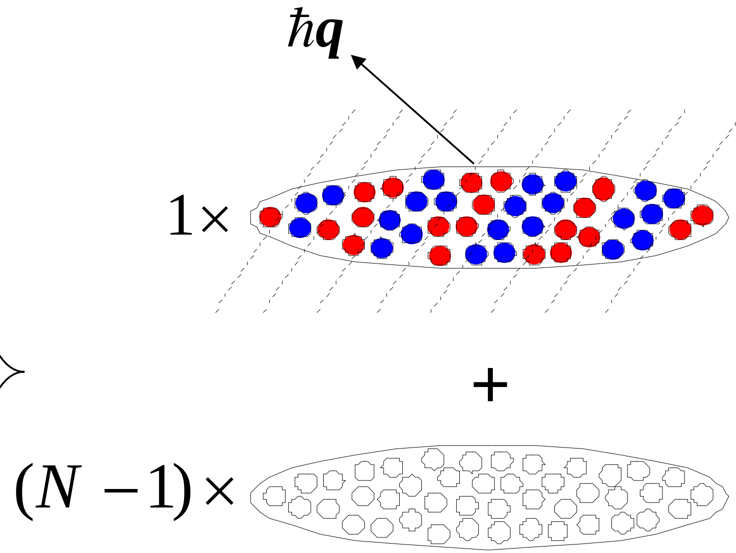




Received 15 May 2006; revised 5 July 2006; accepted 10 July 2006
24 July 2006 / Vol. 14, No. 15 / OPTICS EXPRESS 6912

# The origin of the grating
# (Indiscernability of the atoms )




One atom is excited to the collective
atomic mode defined by $S^+$




$$S^+ |0_a> \equiv \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |g_1, g_2, \cdots, s_i, \cdots, g_{N_a}>$$

# 原子波増幅 (超放射) はフェルミオンでも熱的原子でも起こる

VOLUME 86, NUMBER 19 PHYSICAL REVIEW LETTERS 7 MAY 2001

**Does Matter Wave Amplification Work for Fermions?**

Wolfgang Ketterle and Shin Inouye
*Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
(Received 16 August 2000)

We discuss the relationship between bosonic stimulation, density fluctuations, and matter wave gratings. It is shown that enhanced stimulated scattering, matter wave amplification, and atomic four-wave mixing do not require macroscopic occupation of a single quantum state. These processes are in principle possible for fermionic or nondegenerate samples, if they are prepared in a cooperative state. In practice, there are limitations due to short coherence times.

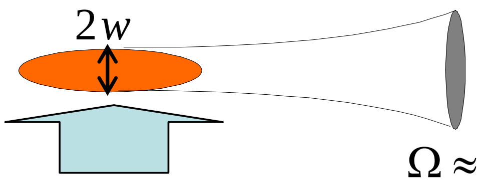
PRL **94**, 083602 (2005) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2005

**Superradiant Light Scattering from Thermal Atomic Vapors**

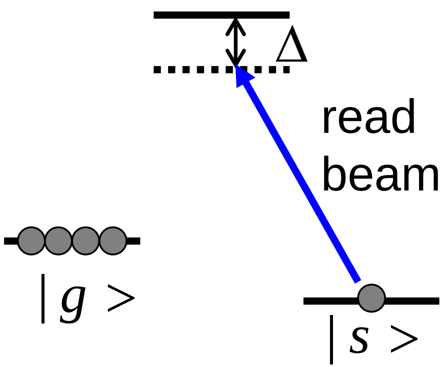
Yutaka Yoshikawa,* Yoshio Torii, and Takahiro Kuga
*Institute of Physics, University of Tokyo, 3-8-1, Meguro-ku, Komaba, Tokyo 153-8902, Japan.*
(Received 12 July 2004; published 4 March 2005)

Superradiant light scattering from noncondensed, thermal atomic vapors was experimentally studied. We found that superradiant gain is independent of quantum degeneracy and determined only by the shape of the atomic cloud and a contained number of atoms. Superradiant pump-probe spectroscopy was also developed to measure the atomic correlation function, revealing the Doppler-width-limited coherence time of the thermal gas and sudden buildup of long-lived coherence below the transition temperature.

# Raman scattering rate for a cigar-shaped atomic ensemble




$$\Omega \approx \left(\frac{\lambda}{w}\right)^2$$




Single-atom Raman scattering rate

$$R = \Gamma \frac{\Omega_{\mathrm{p}}^2}{4\Delta^2}$$

$N_a$-atom Raman scattering rate

$$R_N = f(\theta) R \langle J, M \,|\, J_+ J_- \,|\, J, M \rangle \Omega$$

$$= f(\theta) R N_s (N_g + 1) \Omega$$

Mode field pattern

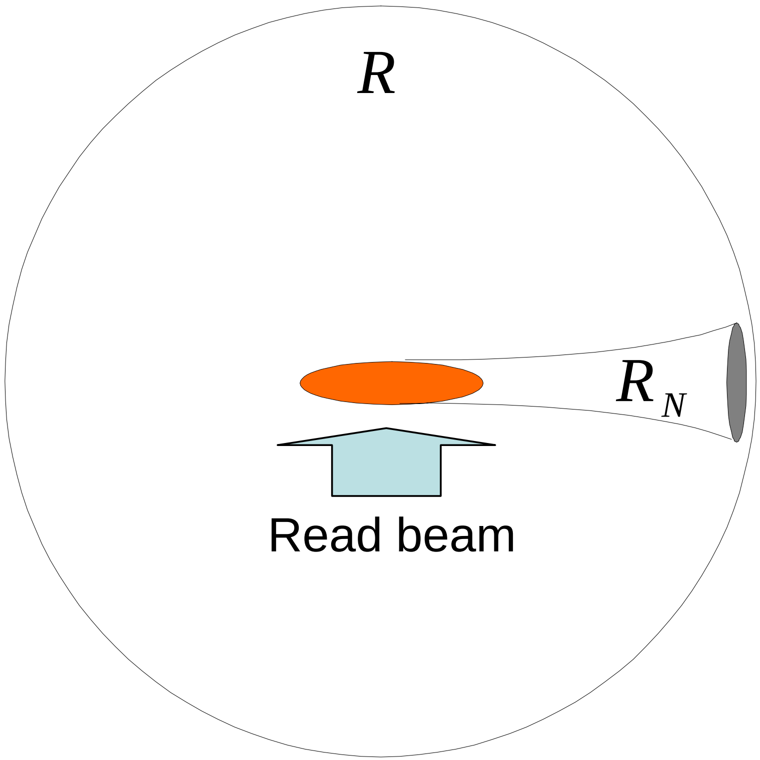
Phase matching solid angle

For the reading ($N_g = N$-1, $N_s = 1$)

$$R_N = \boxed{N} f(\theta) R \, \Omega$$

Collective enhancement

# Spontaneous scattering vs. Collective scattering




The ratio between spontaneous and collective Raman scattering rates:

$$\frac{R_N}{R} = N\eta$$ : cooperativity parameter

$$\eta \equiv f(\theta)\Omega$$ : single-atom optical depth

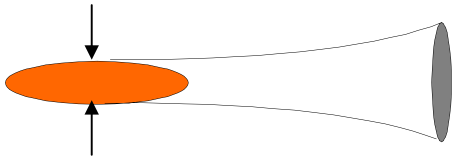
$$\Omega \approx \left(\frac{\lambda}{w}\right)^2 \quad f(\theta) = \begin{cases} \dfrac{3\sin^2\theta}{8\pi} & (\pi\text{-pol.}) \\ \dfrac{3(1+\cos^2\theta)}{16\pi} & (\sigma\text{-pol.}) \end{cases}$$

The probability that an atom in the collective mode emits a photon into the solid angle $\Omega$

$$P_s = \frac{N\eta}{1+N\eta}$$

# Cooperativity parameter of Bose condensates

Typical size of a Bose condensate: $d$ = 10 μm



$$\Omega \approx \left(\frac{\lambda}{d}\right)^2 \approx 10^{-3}$$

Typical number of atoms in a Bose condensate: $N = 10^6$

Cooperativity parameter for a typical Bose condensate:

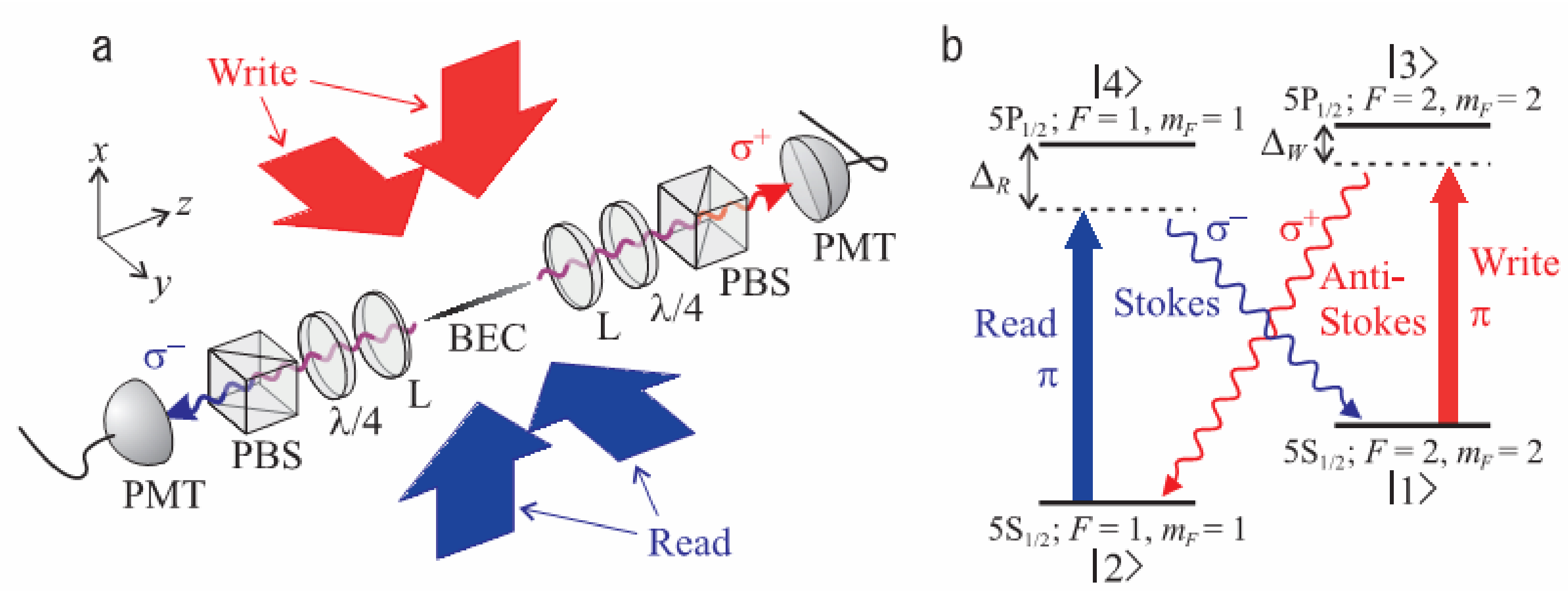
$$N\eta \approx N\Omega \approx 10^3$$

Probability of successful retrieval of a single photon:

$$P_s = \frac{N\eta}{1+N\eta} \approx 99.9\%$$

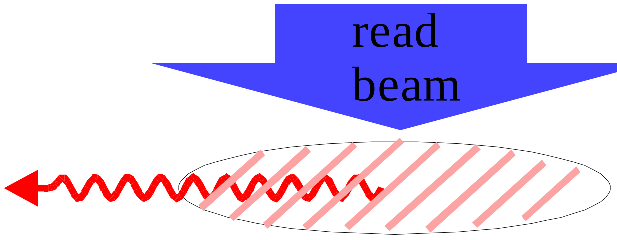
BEC is ideal for storage of a single photon!

# Multiple storage and retrieval of light pulses in a BEC



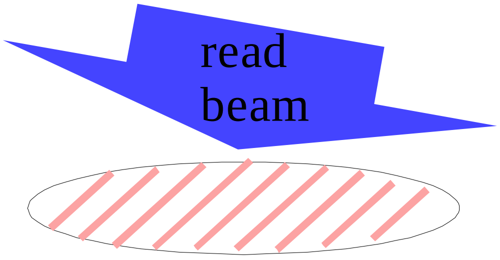
# Selective retrieval of phonons (Phase-matching condition)

Phase-matched read beam



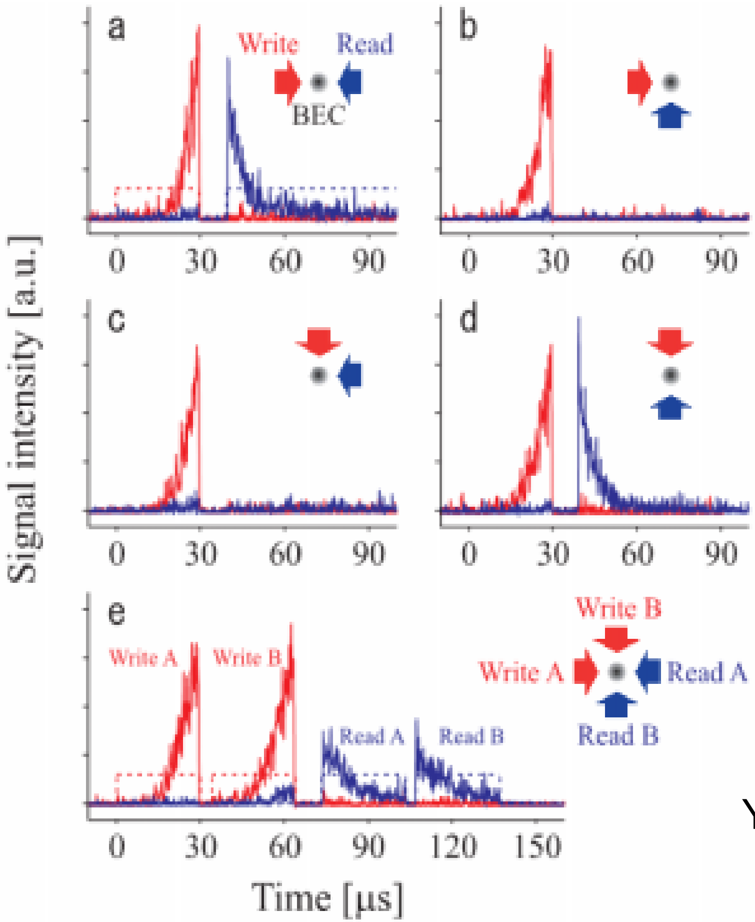

The read beam is diffracted
(successful retrieval)

Phase-mismatched read beam




The read beam just passes
through

# Multiple storage and retrieval of light pulses in a BEC




Possible applications

- Arbitrary-number photon generator
- Multiple quantum memory
- Quantum atom optics

Y. Yoshikawa, K. Nakayama, Y. T. and T. Kuga, Phys. Rev. Lett. **99**, 220407 (2007).

# まとめ

- 共振器がなくとも、十分な光学密度を持つ原子集団を用いれば、特定の電磁場モードに光子を選択的に放出できる

- BECの長いコヒーレンス時間を利用すれば、1つのBEC内に複数の単一光子の情報を書き込み、任意の時間に読み出すことができる。さらに、任意光子発生器も実現できる

→量子光学の新しいツール

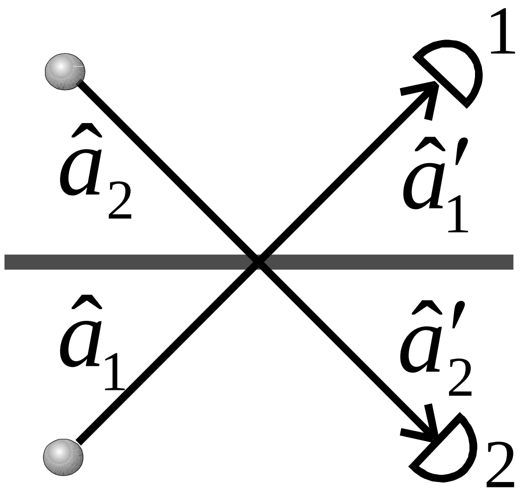
# Two-photon interferance

Input state

$$\hat{a}_1^+ \hat{a}_2^+ \,|\, 0, 0 >$$

Beam splitter operation

$$\begin{pmatrix} \hat{a}_1' \\ \hat{a}_2' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$




Output state

$$\frac{1}{2}\left(\hat{a}_1'^+ + \hat{a}_2'^+\right)\left(\hat{a}_1'^+ - \hat{a}_2'^+\right) | \, 0, 0 >$$

**Bunching**

$$= \frac{1}{2}\left(\left(\hat{a}_1'^+\right)^2 + \left(\hat{a}_2'^+\right)^2\right) | \, 0, 0 > = \frac{1}{\sqrt{2}}\left(| \, 2, 0 > + \, | \, 0, 2 >\right)$$

# Measurement of Subpicosecond Time Intervals between Two Photons by Interference

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A fourth-order interference technique has been used to measure the time intervals between two photons, and by implication the length of the photon wave packet, produced in the process of parametric down-conversion. The width of the time-interval distribution, which is largely determined by an interference filter, is found to be about 100 fs, with an accuracy that could, in principle, be less than 1 fs.

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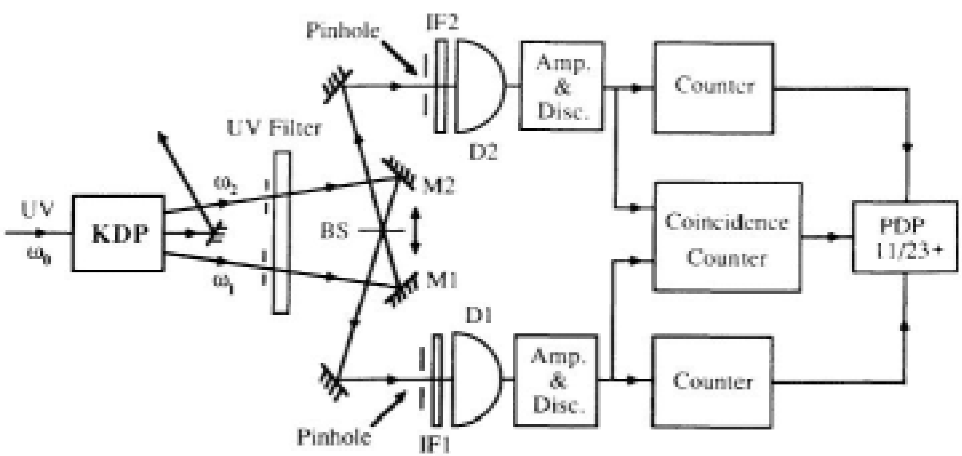


FIG. 1. Outline of the experimental setup.

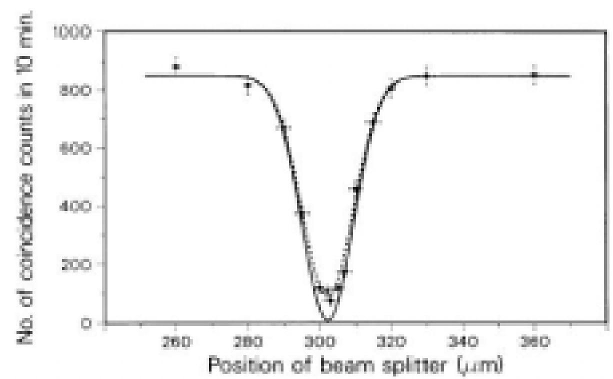


FIG. 2. The measured number of coincidences as a function of beam-splitter displacement $c\delta\tau$, superimposed on the solid theoretical curve derived from Eq. (11) with $R/T=0.95$, $\Delta\omega=3\times10^{13}$ rad $s^{-1}$. For the dashed curve the factor $2RT/(R^2+T^2)$ in Eq. (11) was multiplied by 0.9. The vertical error bars correspond to one standard deviation, whereas horizontal error bars are based on estimates of the measurement accuracy.

# Two-*atom* interference