The magic of Dicke superradiance in a Bose-Einstein condensate: from matter-wave amplification to single-photon storage

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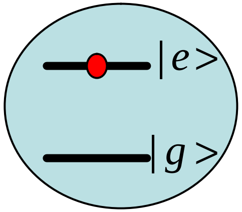
Outline

- What is Dicke superradiance
  - Connection (analogy) between laser and superradiance

- How to make a Bose-Einstein condensate

- Applications of Dicke superradiance in a Bose-Einstein condensate
  - Matter-wave amplification
  - Storage of light (towards quantum memory)

One-atom spontaneous emission



The electric dipole operator

$$\hat{d} = d(s^{+} + s^{-}) \quad \left(s^{+} = |e><g|,\ s^{-} = |g><e|\right)$$

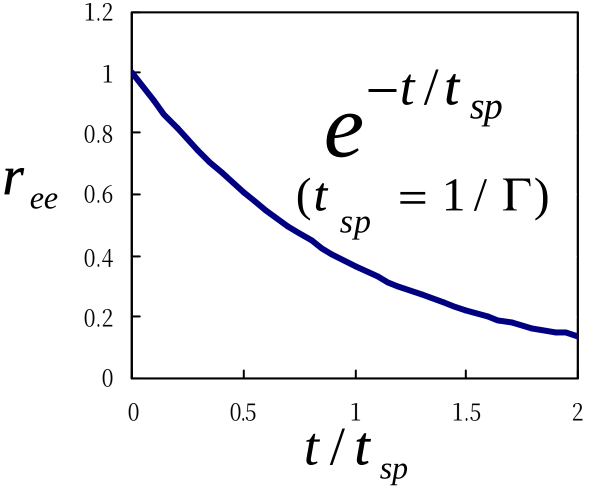
The atom-field interaction Hamiltonian

$$\hat{H}_{\text{int}} = -\hat{d} \cdot \hat{E}_{rad}$$

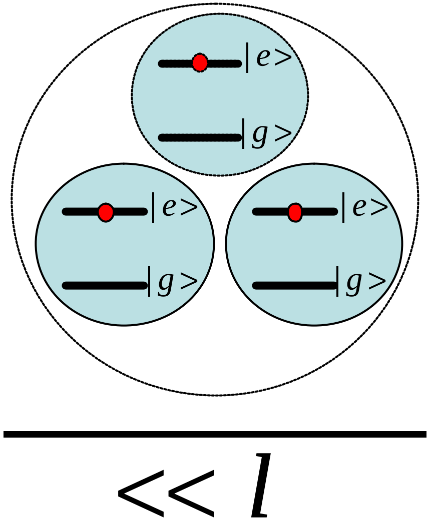
The rate of spontaneous emission

$$R = \Gamma \left\langle s^{+} s^{-} \right\rangle \left(\Gamma = \frac{d^{2} w^{3}}{3p\hbar e_{0} c^{3}} \right)$$

Wigner and Weisskopf (1930)



Three-atom spontaneous emission



The (total) electric dipole operator

$$\hat{d} = d\sum_{i=1}^{3}(s_i^{+} + s_i^{-}) \equiv d(s^{+} + s^{-})$$

$$\left(s^{+} \equiv \sum_{i=1}^{3} | e >_i {}_i< g |,\ s^{+} \equiv \sum_{i=1}^{3} | g >_i {}_i< e | \right)$$

The rate of spontaneous emission

$$R = \Gamma\left\langle s^{+} s^{-} \right\rangle$$

Same form as the single-atom case, but $s^{+}(s^{-})$ is the sum of each raising (lowering) operator

Evolution of the three-atom system

$$|e,e,e>$$

$$R=\Gamma\left\langle s^{+}s^{-}\right\rangle=3\Gamma$$

$$\frac{1}{\sqrt{3}}\left(|g,e,e>+|e,g,e>+|e,e,g>\right)$$

$$R=\Gamma\left\langle s^{+}s^{-}\right\rangle=4\Gamma$$

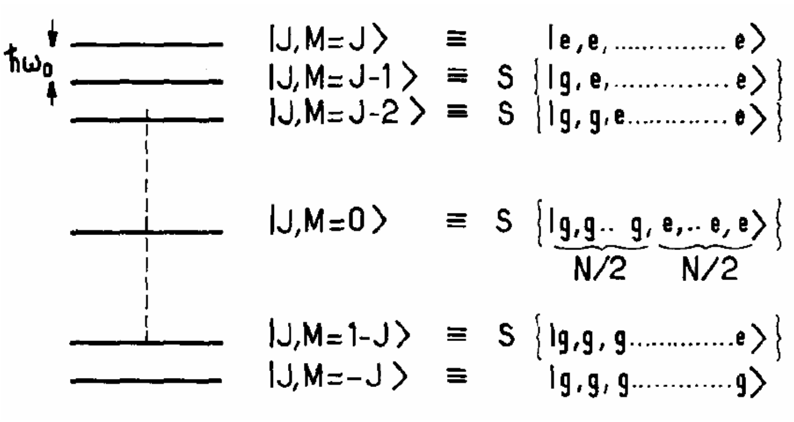
$$\frac{1}{\sqrt{3}}\left(|e,g,g>+|g,e,g>+|g,g,e>\right)$$

$$R=\Gamma\left\langle s^{+}s^{-}\right\rangle=3\Gamma$$

$$|g,g,g>$$

N-atom spontaneous emission

N-atom system ⇔ N spin-1/2 system with the total spin J = N/2 (assumption: *Indiscernability* of the atoms with respect to photon emission)



R. H. Dicke, Phys. Rev. **93**, 99 (1954)

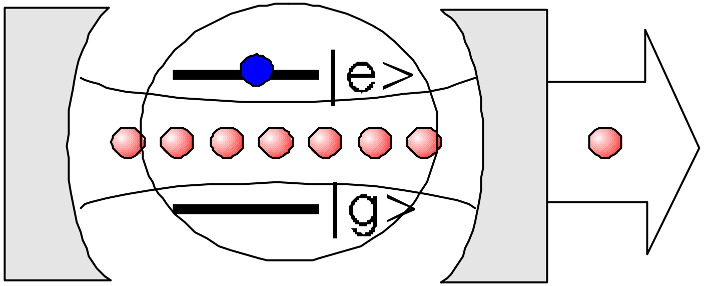
Spontaneous emission rate of the N-atom system:

$$\Gamma_N = \Gamma\langle J,M \,|\, J_+ J_- \,|\, J,M\rangle$$
$$= \Gamma(J+M)(J-M+1)$$
$$= \Gamma N_e (N_g + 1)$$

Enhancement by the number of photons already emitted

Analogy with the laser principle

N photons in the cavity



The interaction Hamiltonian
(after rotating-wave approx.)

$$\hbar g(as^{+}+a^{+}s^{-})$$

$$\left(g=\sqrt{d^{2}w/2e_{0}V}\right)$$

The emission rate of the atom

$$R\propto\left|<g,N-1|(as^{+}+a^{+}s^{-})|e,N>\right|^{2}$$

$$=(N+1)$$

Stimulated emission

Spontaneous emission

LASER: Light Amplification by Stimulated Emission of Radiation

Comparison between ordinary and superradiant emission

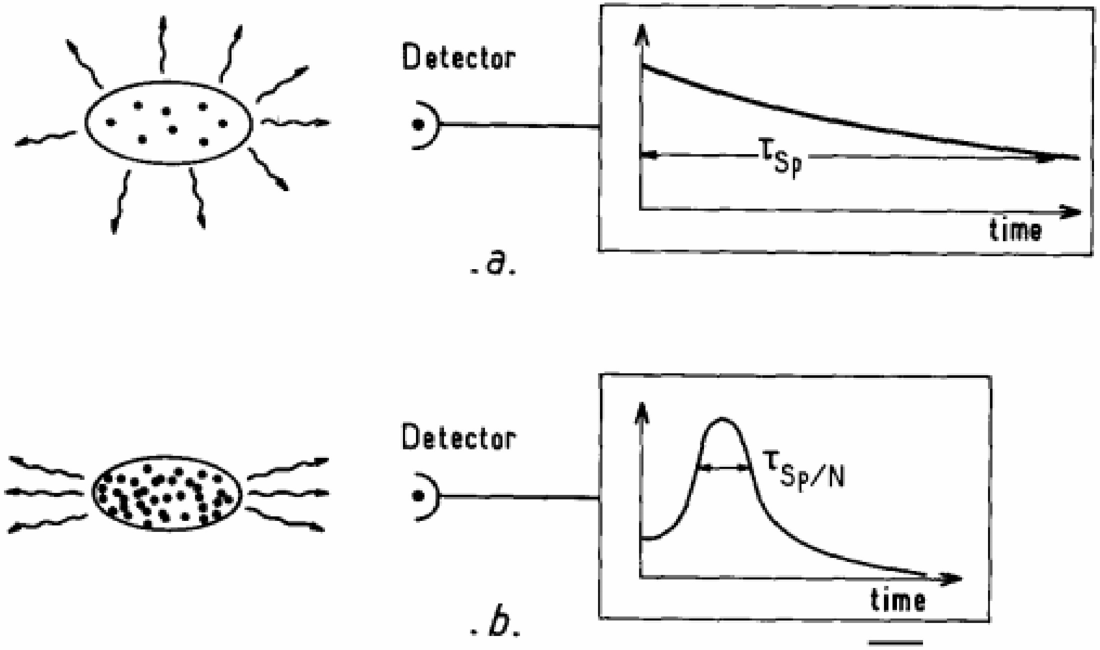
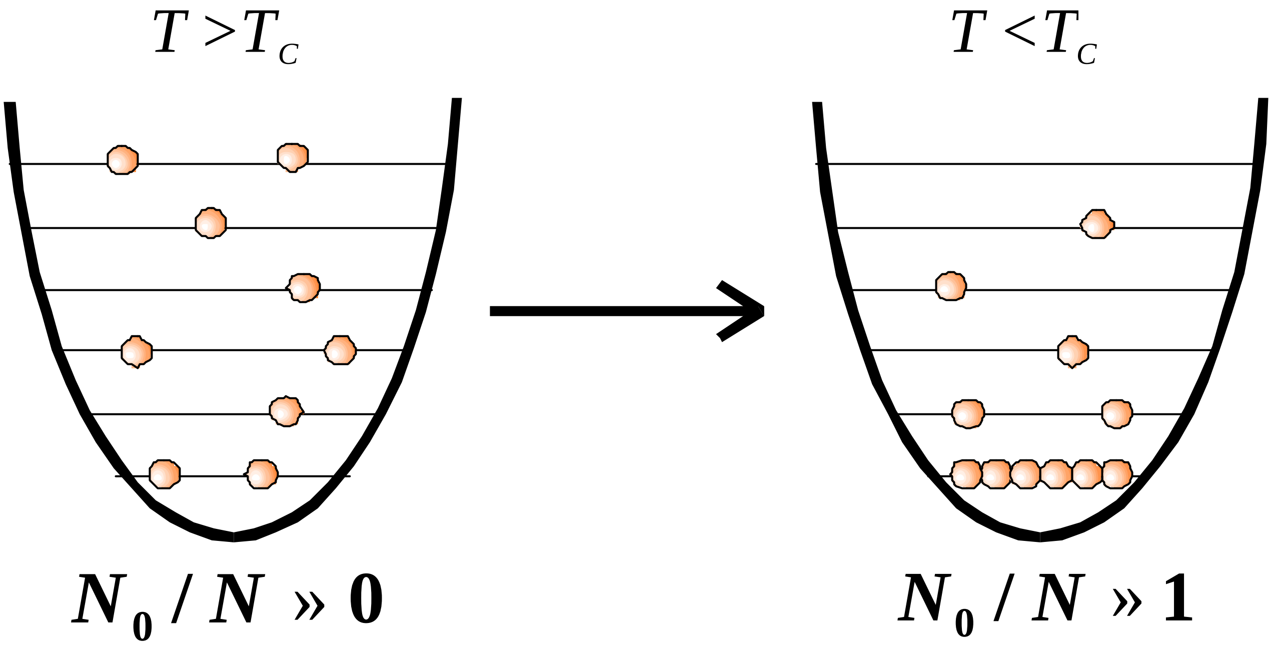


Fig. 1. Comparison between the general characteristics of ordinary fluorescence and superradiance experiments. (a) Ordinary spontaneous emission is essentially isotropic with an exponentially decaying intensity (time constant τ_{sp}). (b) Superradiance is anisotropic with an emission occurring in a short burst of duration $\sim\tau_{sp}/N$.

From M. Gross and S. Haroche, Phys. Rep. **93**, 301 (1982)

What is Bose-Einstin condensation?

Macroscopic occupation of atoms in the lowest quantum state of motion



The criterion of BEC

$$r_{ps} > 2.612$$

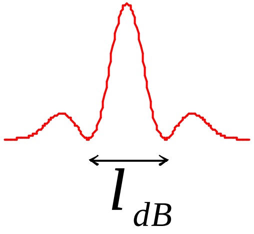
Predicted by
Einstein in 1925

$$r_{ps} \equiv n l_{dB}^{3}$$

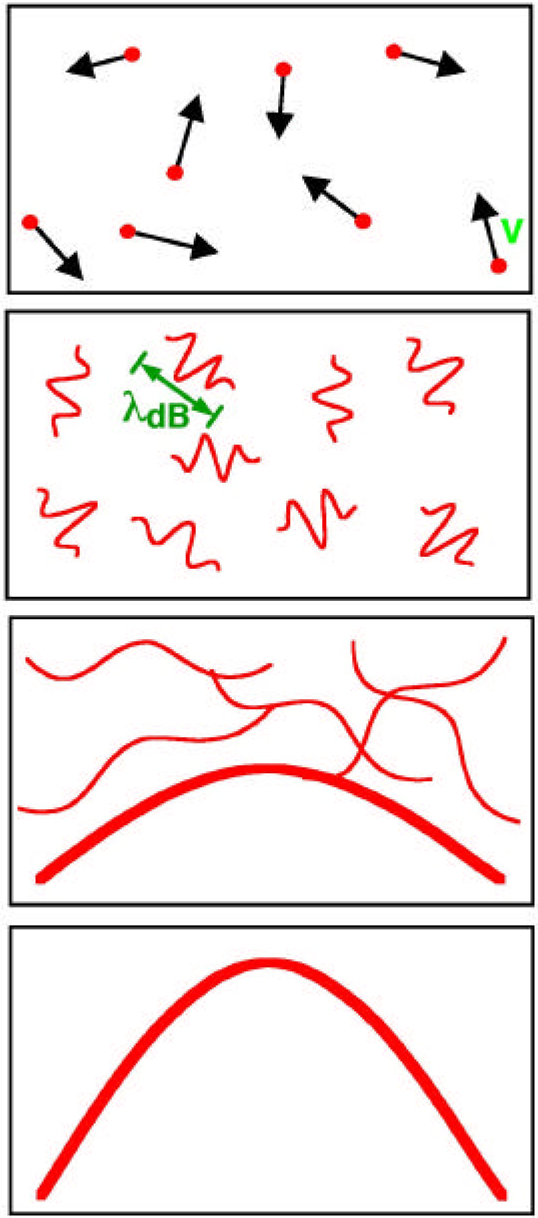
Phase space density (the number of atoms in the lowest quantum state)

$$l_{dB} \equiv \frac{h}{\sqrt{2pmk_B T}}$$

Thermal de Briglie wave length
(the average size of wavepackets)



BEC is formed when the wavepackets overlap with each other !



T∼300K

atoms behave as “billiard balls”

↓ Laser cooling

T∼100 μK

Wave nature begins to manifest

↓ Evaporative cooling

T∼1μK

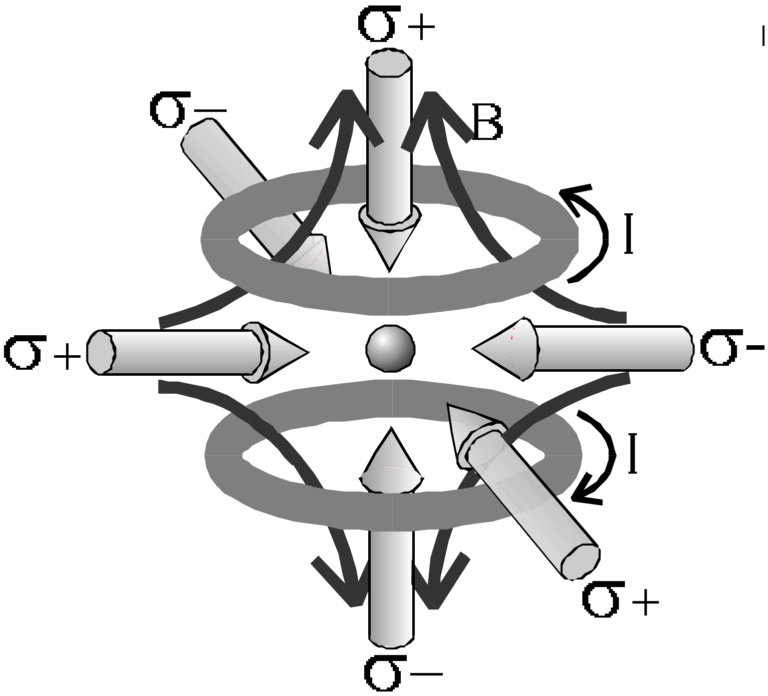
Wavepackets begin to overlap

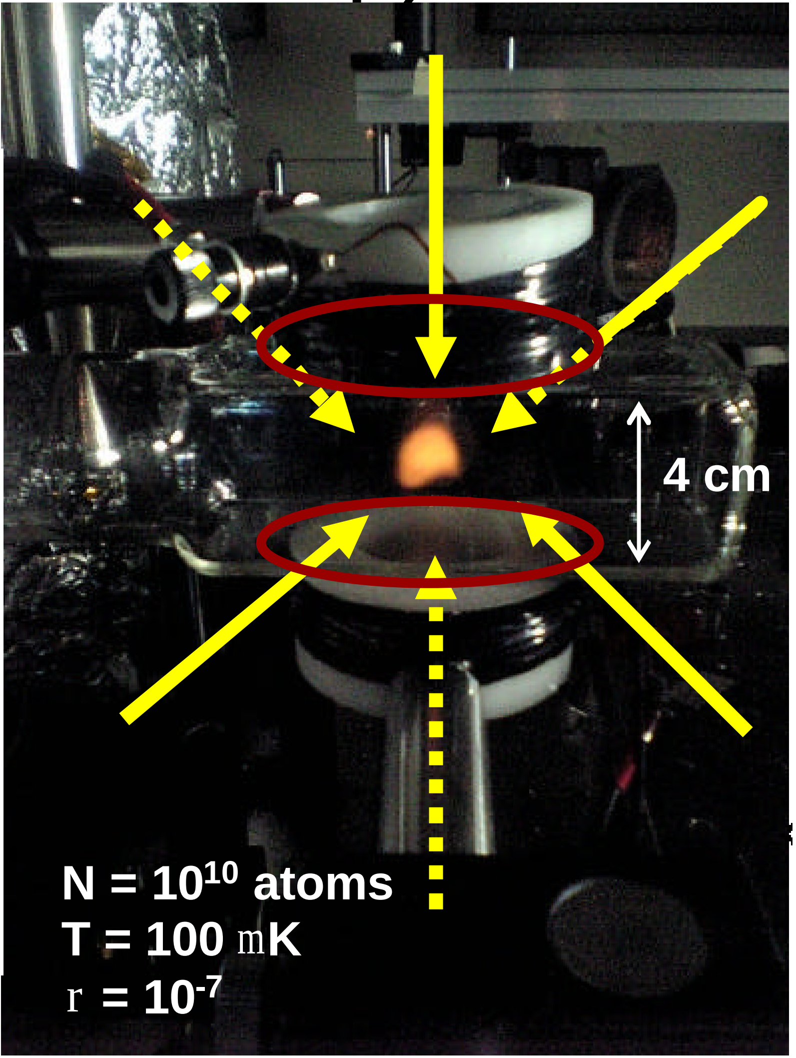
↓ Evaporative cooling

T∼100nK

One giant matter wave
Bose-Einstein condensation

Cooling and Trapping of Rb atoms (Magneto-optical trap)





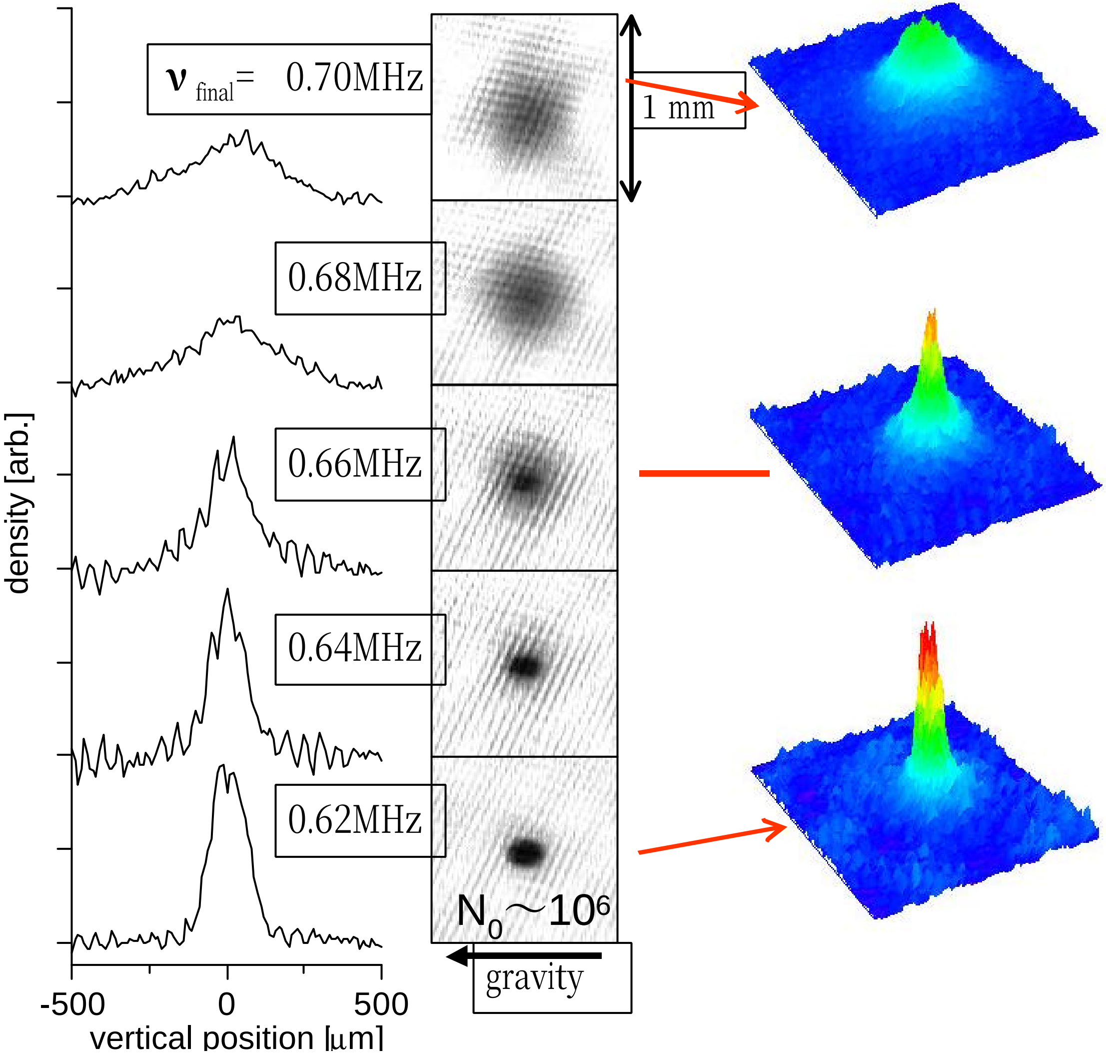
N = 10^{10} atoms
T = 100 mK
r = 10^{-7}

What is evaporative cooling?

Nice applet at Colorado University Website

http://www.colorado.edu/physics/2000/applets/bec.html

Phase transition from a thermal cloud to a Bose condensate



The properties of BEC

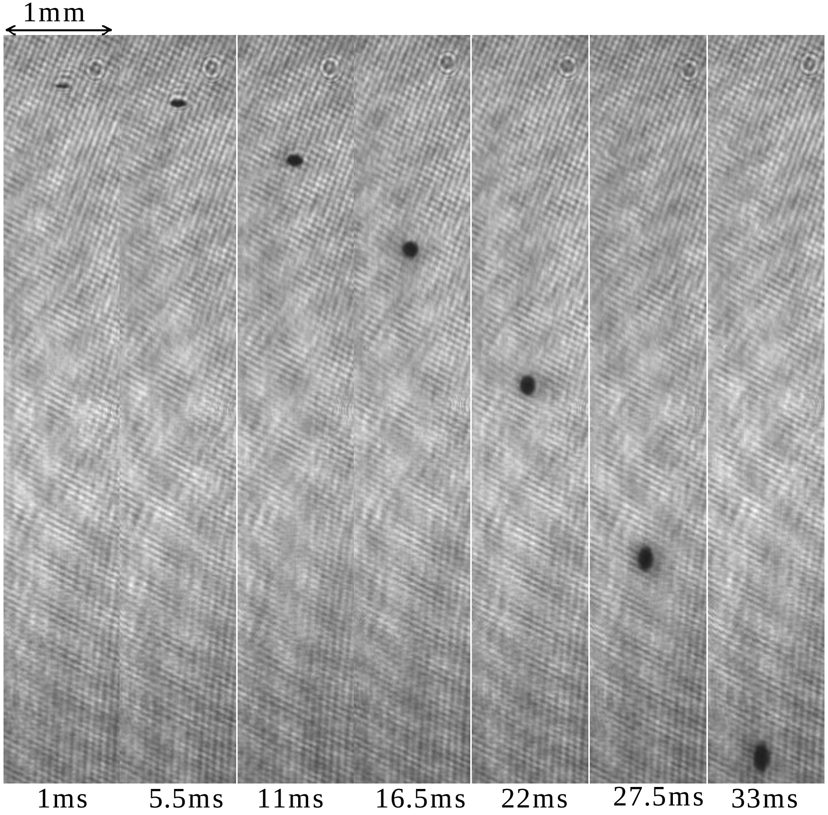
1. **narrow velocity width below the recoil velocity （6 mm/s for Rb87）**

2. **Well localized in space （10 μm～100 μm）**

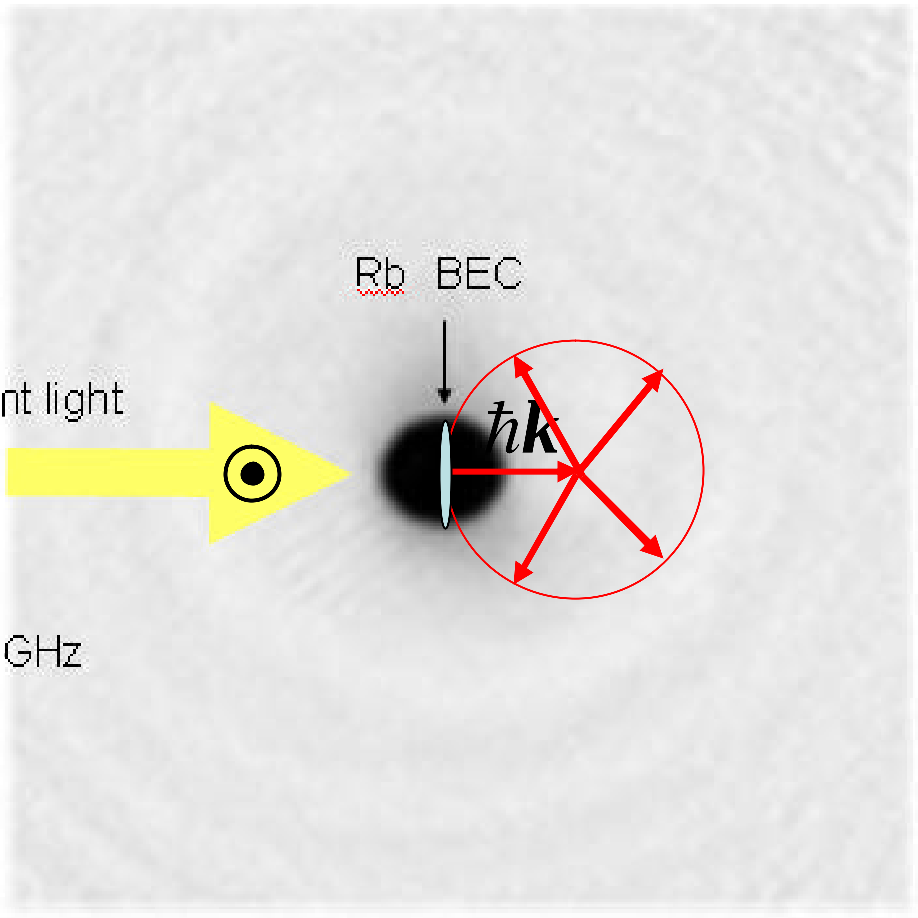
3. **Spatial density of ～10^{14} atoms/cm^3**

4. **Coherent**

Images of a BEC released from the magnetic trap



Rayleigh scattering in a Rb BEC



Off-resonant light

63 mW/cm^2

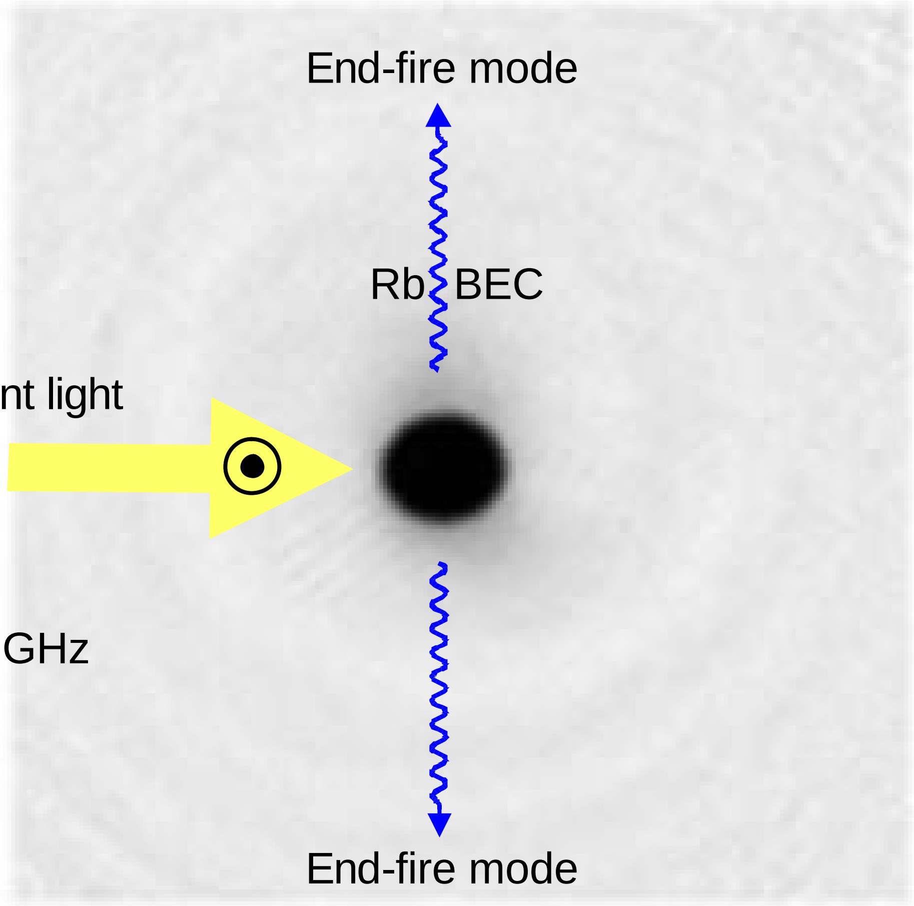
detuning: -4.4 GHz

Pulse duration:
25 μs-3200 μs

30 ms TOF

D. Schneble, Y.T., M. Boyd, E. W. Streed, D. E. Pritchard, and W. Ketterle, Science **300**, 475 (2003)

Rayleigh scattering in a Rb BEC



End-fire mode

Rb BEC

Off-resonant light

63 mW/cm^2

detuning: -4.4 GHz

End-fire mode

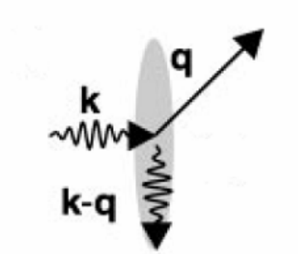
30 ms TOF

Pulse duration:
25 μs-3200 μs

D. Schneble, Y.T., M. Boyd, E. W. Streed, D. E. Pritchard, and W. Ketterle, Science **300**, 475 (2003)

Fully-quantum picture
(Fermi's Golden Rule)

Scattering process:



Condensate

Recoiling atoms

$$|N_0, n_k; N_q, n_{k-q}> \xrightarrow[\hat{H}=\hat{a}_q^+\hat{c}_{k-q}^+\hat{a}_0\hat{c}_k]{} |N_0-1, n_k-1; N_q+1, n_{k-q}+1>$$

Incident photon

Scattered photon

Scattering rate:

$$W \propto |< N_0-1, n_k-1; N_q+1, n_{q-k}+1|\hat{H}|N_0, n_k; N_q, n_{k-q}>|^2$$

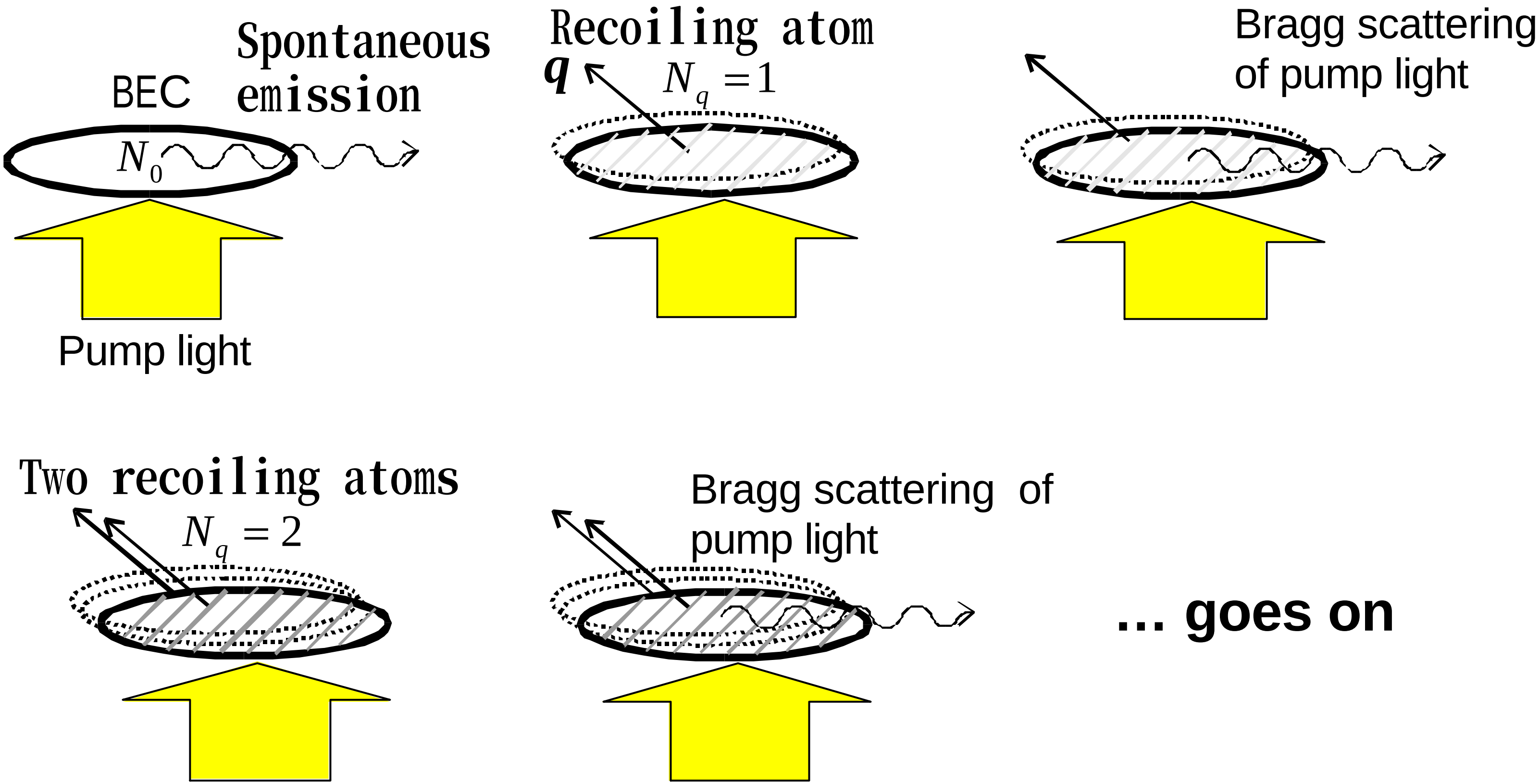
$$= N_0 n_k (N_q+1)(\cancel{n_{k-q}}+1) \propto N_0(N_q+1)$$

neglect

Stimulated scattering (Bosonic enhancement)

Spontaneous scattering

Semi-classical interpretation of superradiance



BEC

Spontaneous emission

N_0

Pump light

Recoiling atom

$\boldsymbol{q}$

$N_q = 1$

Bragg scattering of pump light

Two recoiling atoms

$N_q = 2$

Bragg scattering of pump light

… goes on

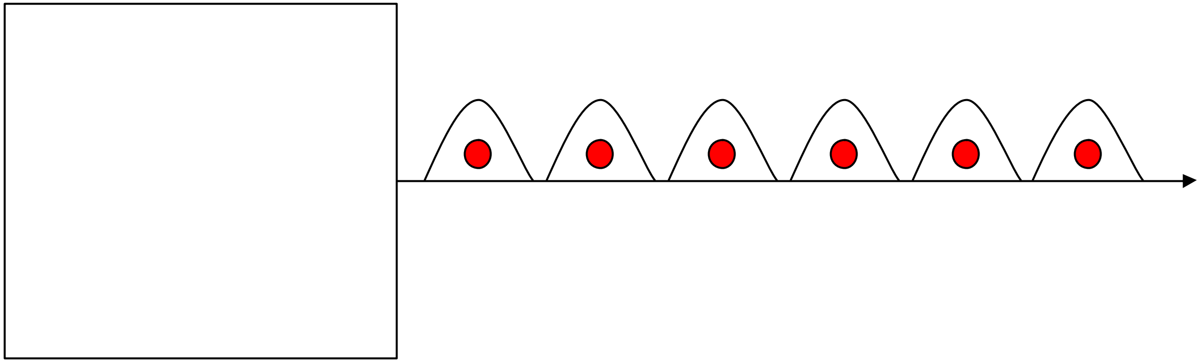
The rate of light scattering is enhanced

by the number of recoiling atoms

$$\dot{N}_q \propto N_0 (N_q + 1)$$

Lasing of matter-wave

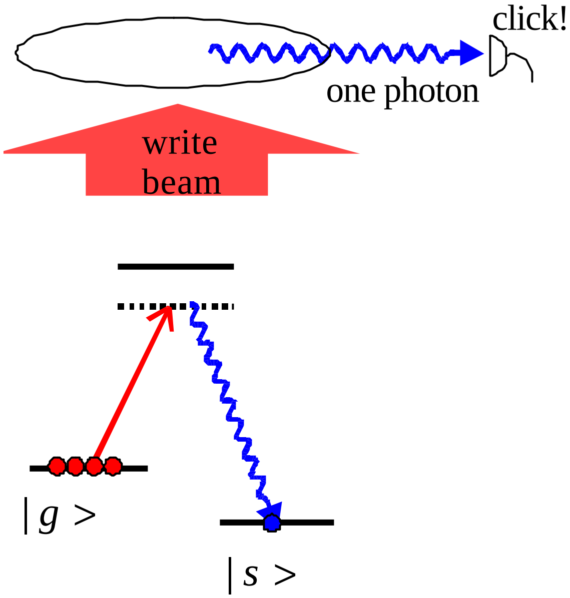
Ideal single-photon generator



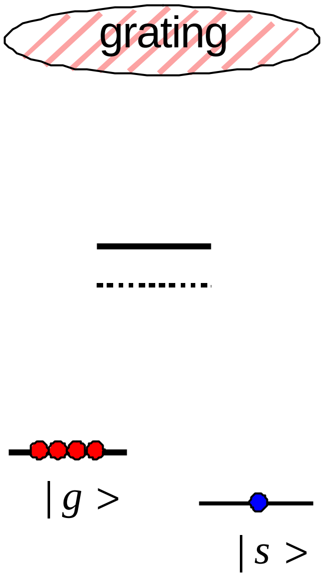
A device which produces light pulses containing only one photon in a well-defined mode

Writing, storing, and reading of a single photon

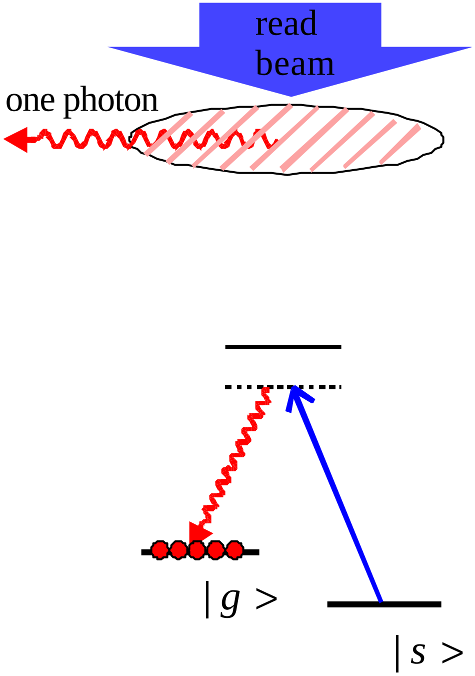
writing



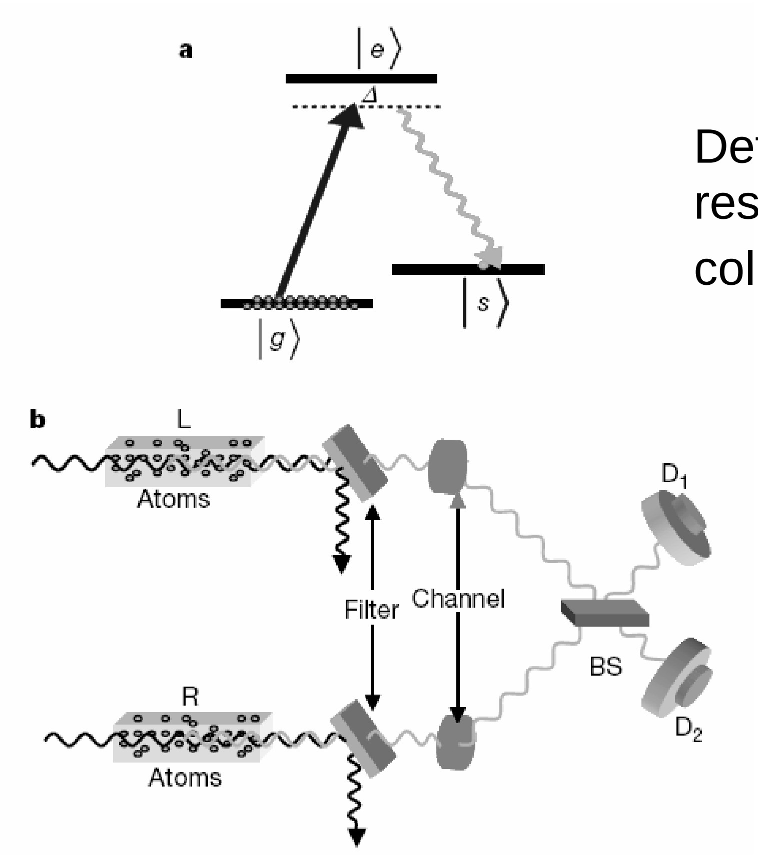
storing



reading



Motivation: DLCZ protocol

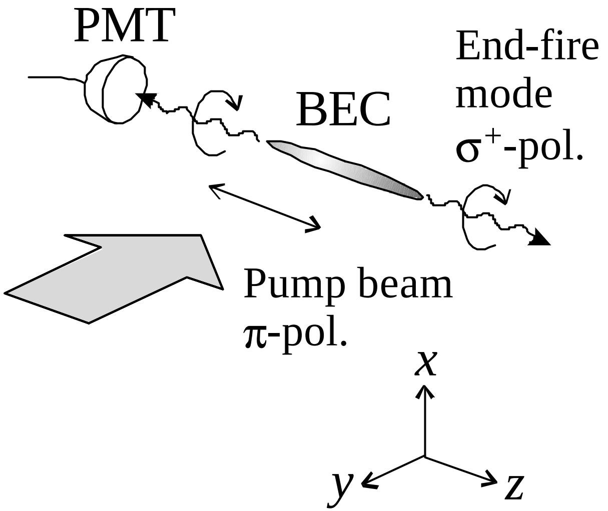


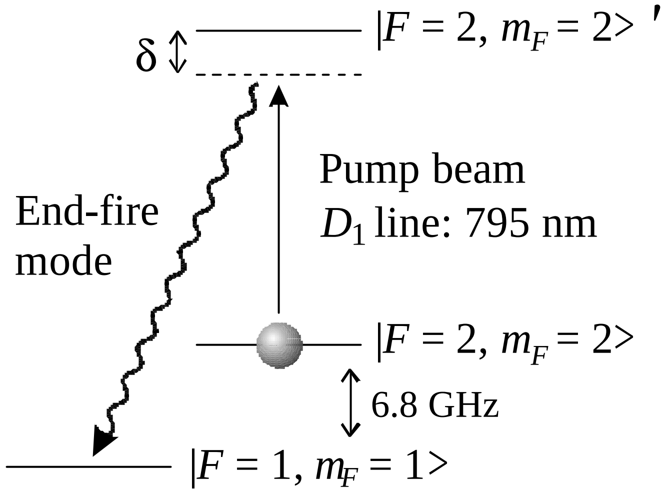
Detection of a forward-scattered photon results in the excitation of the symmetric collective mode defined by

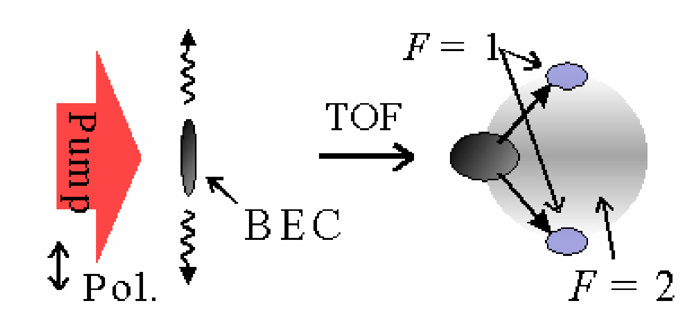
$$S^{+} \equiv \frac{1}{\sqrt{N}} \sum_{i=1}^{N} | s >_i < g |$$

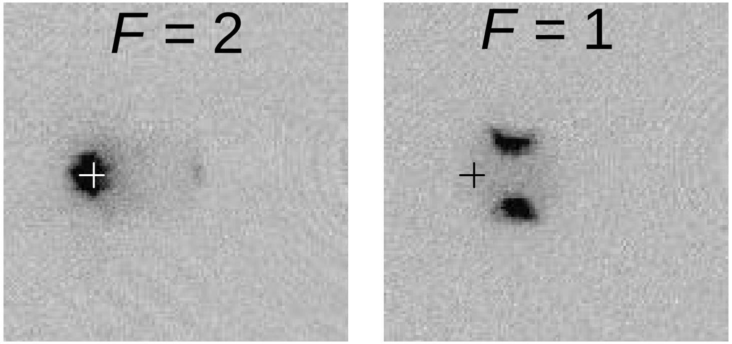
L.-M. Duan, M. D. Lukin, J. I. Cirac,and P. Zoller, Nature. **414**, 413 (2001)

Superradiant Raman scattering in a Bose condensate



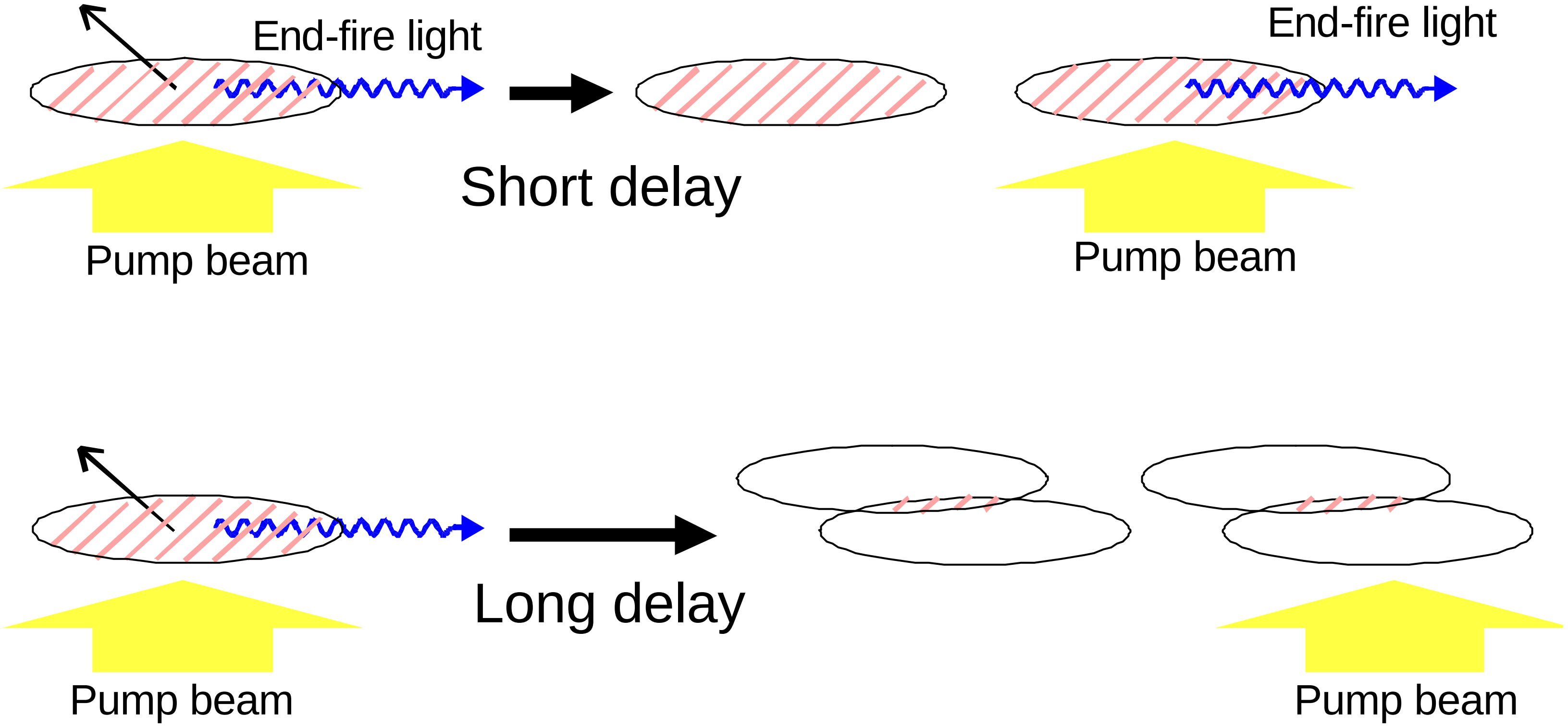






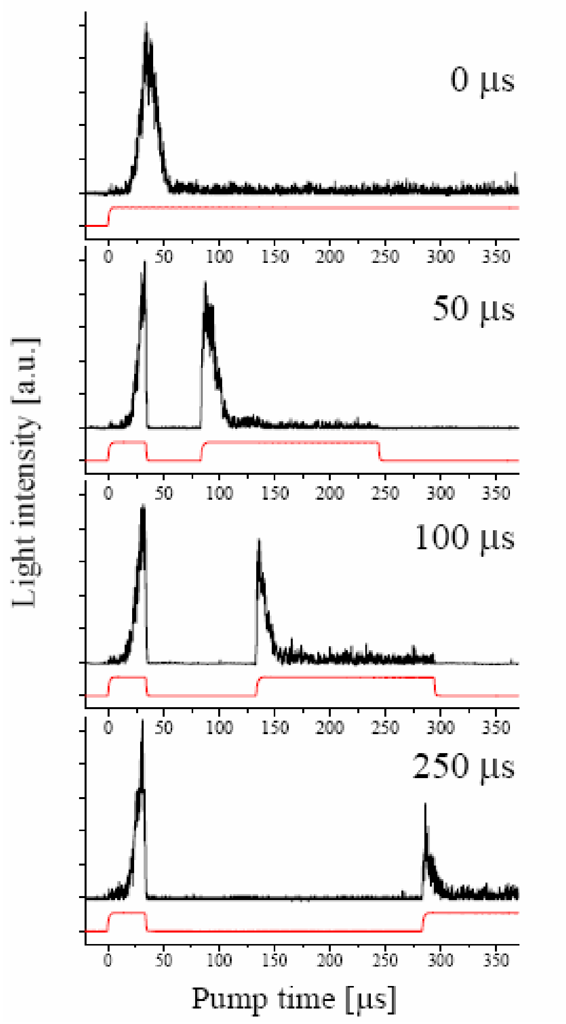
Y. Yoshikawa, T. Sugiura, Y. T., and T. Kuga, PRA **69** 041603 (2004)

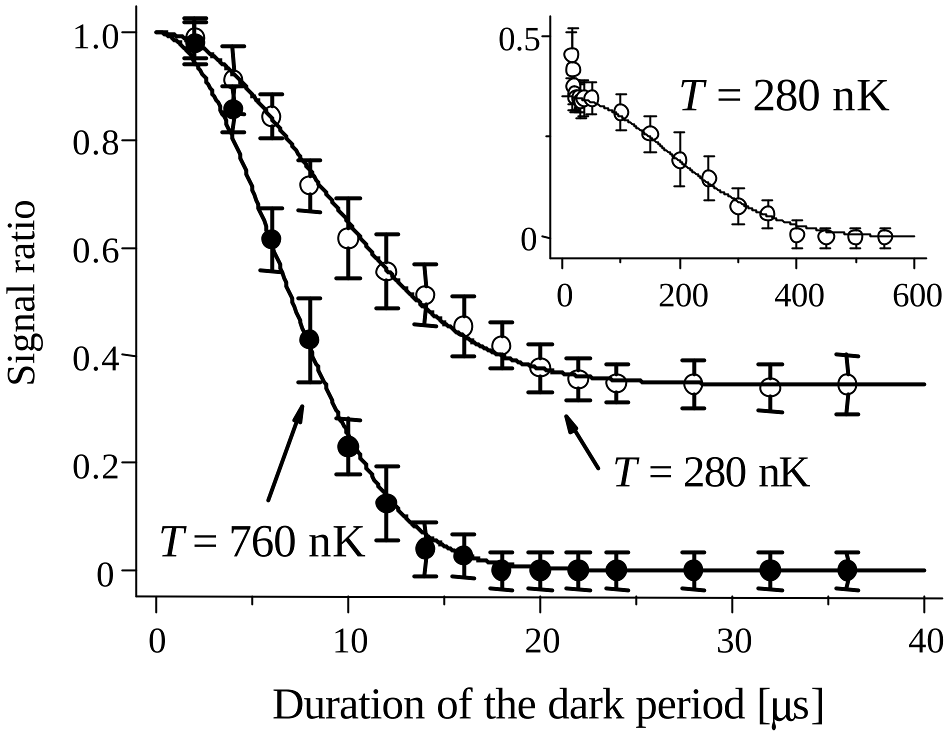
How long does the grating survive?



Storage (coherent) time of the grating is limited by the size of the wavepacket

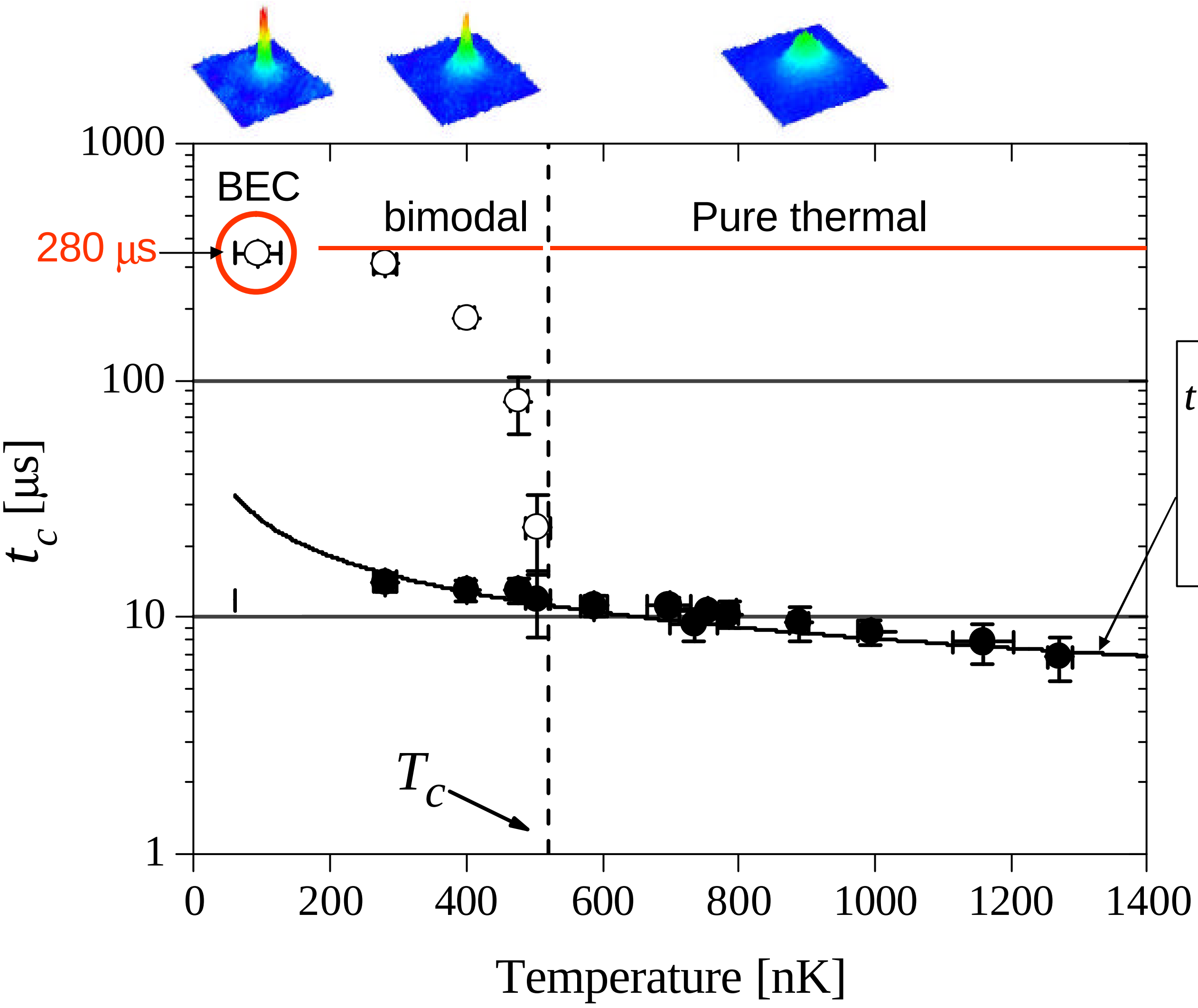
Storage (coherence) time measurement





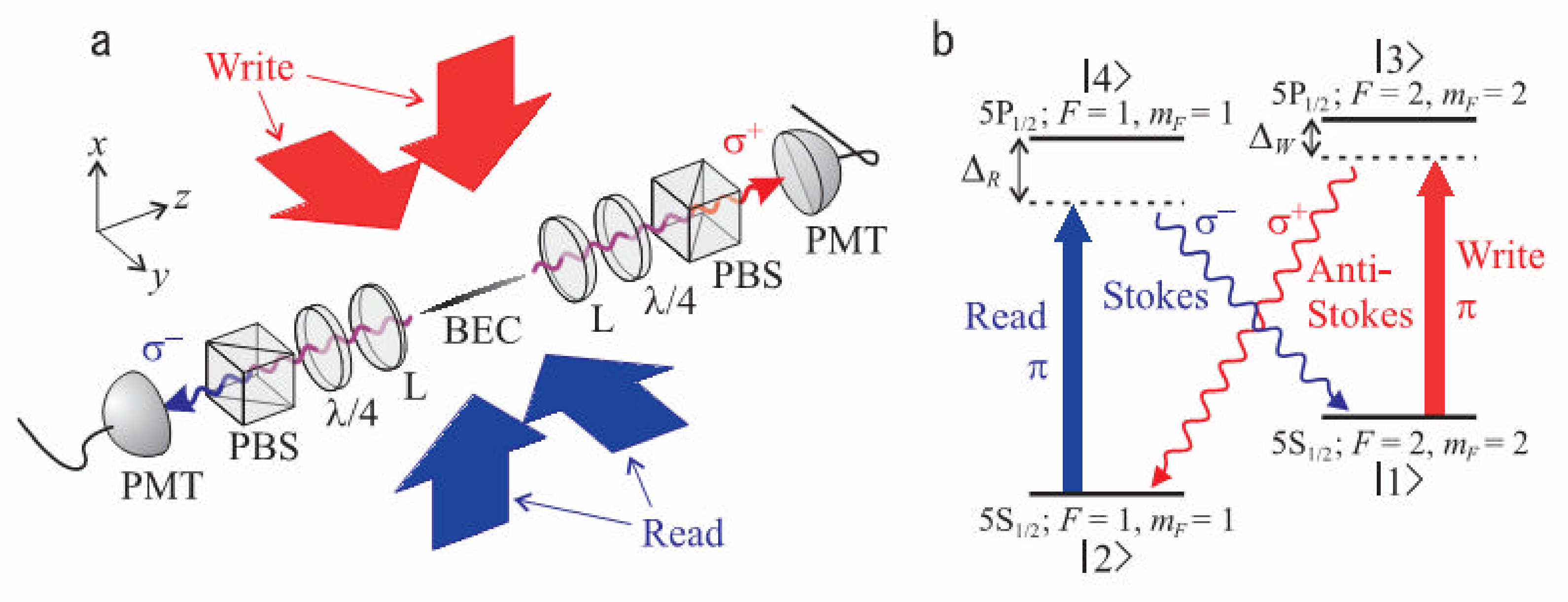
Y. Yoshikawa, Y. T. and T. Kuga, PRL **94** 083602 (2005)

Storage time vs. temperature



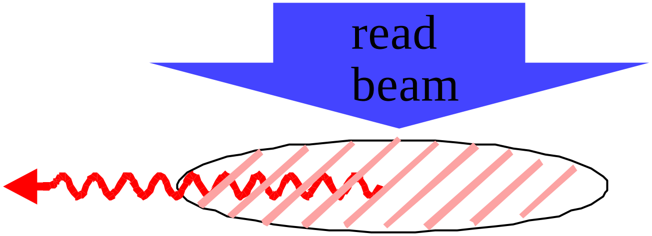
$$t_c = \frac{1}{2k\bar{v}} = \frac{l}{4p}\sqrt{\frac{m}{k_B T}}$$

Multiple storage and retrieval of light pulses in a BEC



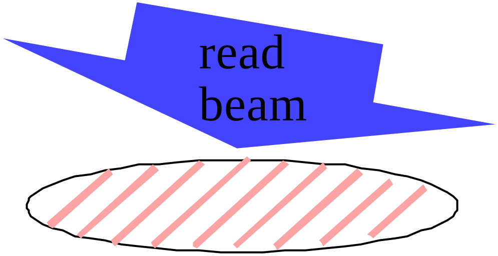
Selective retrieval of phonons
(Phase-matching condition)

Phase-matched read beam



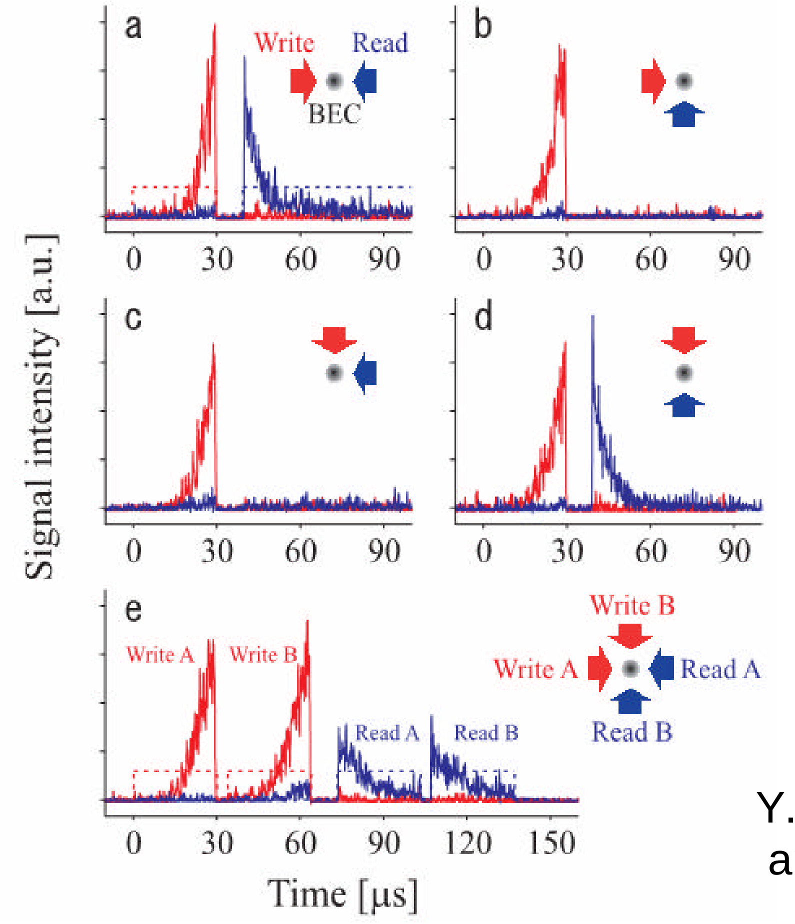
The read beam is diffracted
(successful retrieval)

Phase-mismatched read beam



The read beam just passes
through

Multiple storage and retrieval of light pulses in a BEC



Possible applications

- Arbitrary-number photon generator
- Multiple quantum memory
- Quantum atom optics

Y. Yoshikawa, K. Nakayama, Y. T. and T. Kuga, arXiv:physics.atom-ph/0706.1821

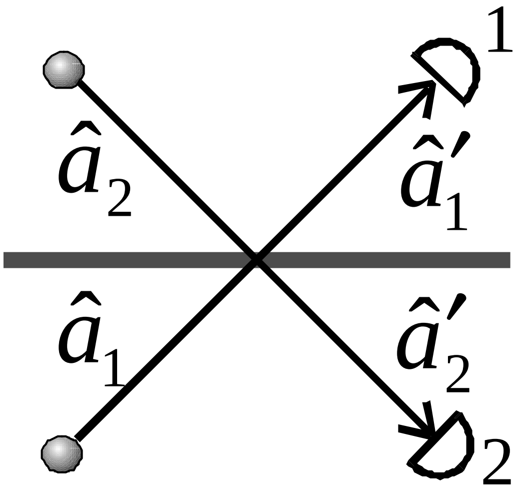
Two-photon interferance

Input state

$$\hat{a}_1^+ \hat{a}_2^+ \,|\,0,0>$$

Beam splitter operation

$$\begin{pmatrix} \hat{a}_1' \\ \hat{a}_2' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$



Output state

$$\frac{1}{2}\left(\hat{a}_1'^{+} + \hat{a}_2'^{+}\right)\left(\hat{a}_1'^{+} - \hat{a}_2'^{+}\right)|\,0,0>$$

Bunching

$$= \frac{1}{2}\left(\left(\hat{a}_1'^{+}\right)^2 + \left(\hat{a}_2'^{+}\right)^2\right)|\,0,0> = \boxed{\frac{1}{\sqrt{2}}\left(|\,2,0> + |\,0,2>\right)}$$

VOLUME 59, NUMBER 18 PHYSICAL REVIEW LETTERS 2 NOVEMBER 1987

Measurement of Subpicosecond Time Intervals between Two Photons by Interference

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(Received 10 July 1987)

A fourth-order interference technique has been used to measure the time intervals between two photons, and by implication the length of the photon wave packet, produced in the process of parametric down-conversion. The width of the time-interval distribution, which is largely determined by an interference filter, is found to be about 100 fs, with an accuracy that could, in principle, be less than 1 fs.

PACS numbers: 42.50.Bs, 42.65.Re

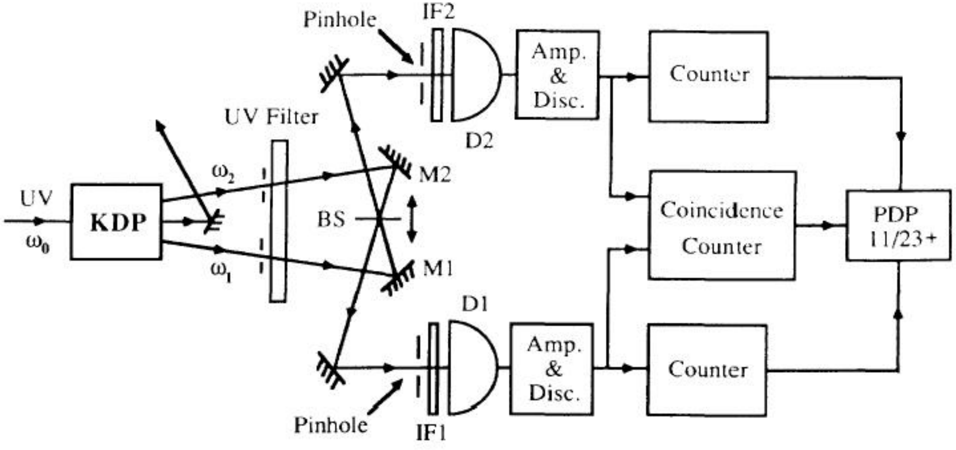


FIG. 1. Outline of the experimental setup.

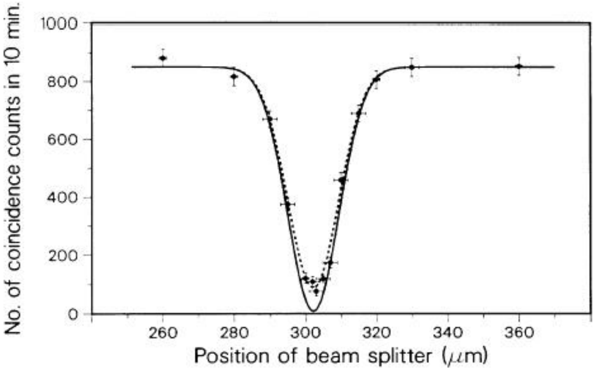
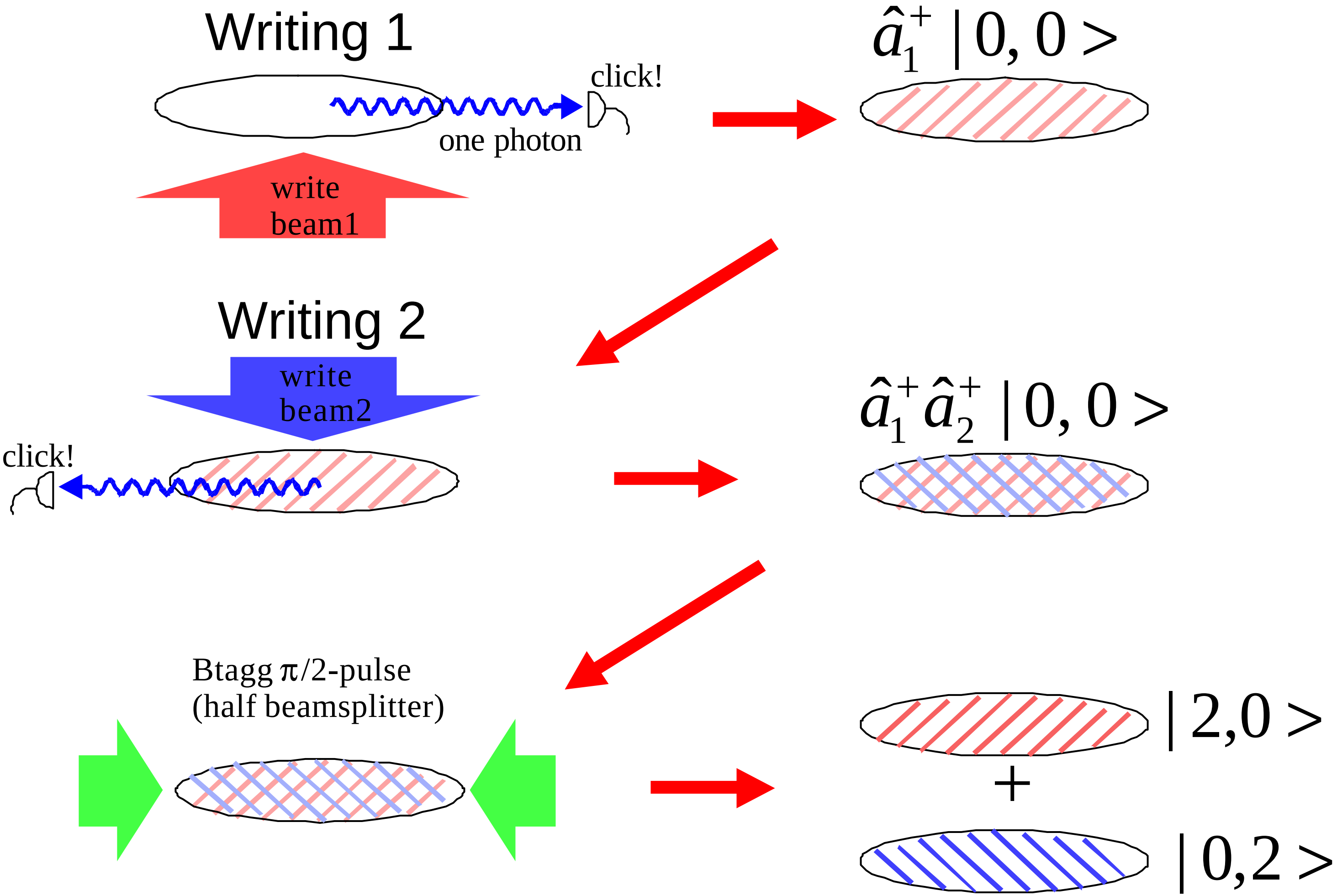


FIG. 2. The measured number of coincidences as a function of beam-splitter displacement $c\,\delta\tau$, superimposed on the solid theoretical curve derived from Eq. (11) with $R/T=0.95$, $\Delta\omega=3\times10^{13}$ rad s^{-1}. For the dashed curve the factor $2RT/(R^2+T^2)$ in Eq. (11) was multiplied by 0.9. The vertical error bars correspond to one standard deviation, whereas horizontal error bars are based on estimates of the measurement accuracy.

Two-*atom* interference



Writing 1

click!

one photon

write beam1

$\hat{a}_1^+ \,|\, 0, 0 >$

Writing 2

write beam2

click!

$\hat{a}_1^+ \hat{a}_2^+ \,|\, 0, 0 >$

Btagg π/2-pulse
(half beamsplitter)

$|\, 2,0 >$

$+$

$|\, 0,2 >$

Conclusion

- The concept of Dicke superradiance is applicable not only to spontaneous emission but also light scattreing (Rayleigh or Raman)

- Superradiand Rayleigh/Raman scattering offers us new and interesting phenomena and application such as matter-wave amplification and single-photon storage